Calculus 1 — Exam 1
MAT 250, Spring 2017 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -3+} f(x) = -\infty$

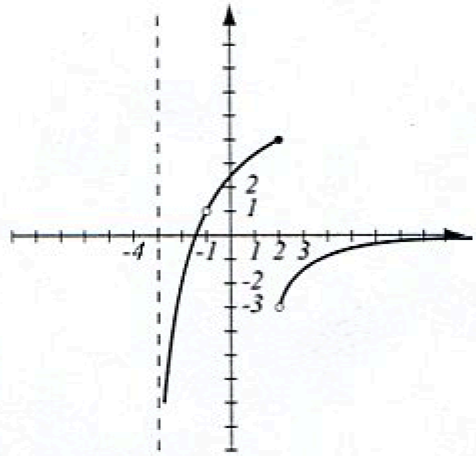
$\lim_{x\to 2^-} f(x) = 4$

$\lim_{x\to 2^+} f(x) = -3$

$\lim_{x\to 2} f(x) =$ dne since one sided limits are diffent

$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to \infty} f(x) = 0$



List points where f is not continuous and justify why it is not continuous at those points.

f is not cont at $x=-1$ since $f(-1)$ is not defined

f is not cont. at $x=2$ since $\lim_{x\to 2} f(x)$ dne

2. (6pts) Let $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3} g(x) = -1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 3} \sqrt{x^3 f(x) - \frac{10}{g(x)}} = \sqrt{\lim_{x\to 3}\left(x^3 f(x) - \frac{10}{g(x)}\right)} = \sqrt{\lim_{x\to 3} x^3 f(x) - \lim_{x\to 3} \frac{10}{g(x)}}$$

$$= \sqrt{\lim_{x\to 3} x^3 \cdot \lim_{x\to 3} f(x) - \frac{\lim_{x\to 3} 10}{\lim_{x\to 3} g(x)}} = \sqrt{27 \cdot 2 - \frac{10}{-1}} = \sqrt{64} = 8$$

3. (10pts) Find $\lim_{x\to 0} x^2 \cdot \sqrt{7 + \sin\left(\frac{1}{x}\right)}$. Use the theorem that rhymes with honey-producing insects.

$$-1 \le \sin\frac{1}{x} \le 1$$

$$6 \le 7 + \sin\frac{1}{x} \le 8 \quad | \sqrt{\ }$$

$$\sqrt{6} \le \sqrt{7 + \sin\frac{1}{x}} \le \sqrt{8}$$

$$\sqrt{6}x^2 \le x^2\sqrt{7 + \sin\frac{1}{x}} \le \sqrt{8}x^2$$

$$\underbrace{\lim_{x\to 0} \sqrt{6}x^2 = 0 \quad \lim_{x\to 0} \sqrt{8}x^2 = 0}$$

same, so by the squeeze theorem,

$$\lim_{x\to 0} x^2\sqrt{7 + \sin\frac{1}{x}} = 0$$

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x\to 5} \dfrac{x^2-5x}{x^2-3x-10} = \lim_{x\to 5} \dfrac{x\cancel{(x-5)}}{\cancel{(x-5)}(x+2)} = \lim_{x\to 5} \dfrac{x}{x+2} = \dfrac{5}{5+2} = \dfrac{5}{7}$

$\dfrac{25-25}{25-15-10} = \dfrac{0}{0}$

5. (7pts) $\lim_{x\to 2} \dfrac{3-\sqrt{x+7}}{x-2} = \lim_{x\to 2} \dfrac{3-\sqrt{x+7}}{x-2} \cdot \dfrac{3+\sqrt{x+7}}{3+\sqrt{x+7}} = \lim_{x\to 2} \dfrac{9-(x+7)}{(x-2)(3+\sqrt{x+7})}$

$\dfrac{3-\sqrt{9}}{2-2}$

$= \lim_{x\to 2} \dfrac{\cancel{2-x}\,(-1)}{\cancel{(x-2)}(3+\sqrt{x+7})} = \lim_{x\to 2} \dfrac{-1}{3+\sqrt{x+7}} = \dfrac{-1}{3+\sqrt{9}} = -\dfrac{1}{6}$

$\dfrac{0}{0}$

6. (7pts) $\lim_{x\to 0} \dfrac{\tan(2x)}{x} = \lim_{x\to 0} \dfrac{\frac{\sin(2x)}{\cos(2x)}}{x} = \lim_{x\to 0} \dfrac{\sin(2x)}{\cos(2x)} \cdot \dfrac{1}{x} \cdot \dfrac{2}{2}$

$= \lim_{x\to 0} \underbrace{\dfrac{\sin(2x)}{2x}}_{\to 1} \cdot \dfrac{2}{\cos(2x)} = 1 \cdot \dfrac{2}{1} = 2$

7. (7pts) $\lim_{x\to\infty} \dfrac{3x^2-5x+4}{x^3-4x^2+x-7} = \lim_{x\to\infty} \dfrac{\cancel{x^2}\left(3-\overbrace{\frac{5}{x}}^{\to 0}+\overbrace{\frac{4}{x^2}}^{\to 0}\right)}{x\,\cancel{x^3}\left(1-\underbrace{\frac{4}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}-\underbrace{\frac{7}{x^3}}_{\to 0}\right)}$

$= \left(\lim_{x\to\infty} \dfrac{1}{x}\right) \cdot \dfrac{3-0+0}{1-0+0-0} = 0 \cdot 3 = 0$

8. (6pts) $\lim_{x\to 2^+} \dfrac{x-6}{4-2x} = \dfrac{-4}{0-} = \infty$

if $x > 2$

$-2x < -4$

$4-2x < 0$

$\left(\dfrac{-4}{\text{small neg}} = \text{large pos.}\right)$

9. (14pts) The equation $x^2 + 4x = 2^x + 5$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 3)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

a) $x^2 + 4x = 2^x + 5$

$\underbrace{x^2 + 4x - 2^x}_{f(x),\ \text{is continuous}} = 5$

$f(0) = -1 < 5$

$f(3) = 9 + 12 - 8 = 13 > 5$

Since $f(0) < 5 < f(3)$,

by IVT there is a c in $(0,3)$

s.t. $f(c) = 5$

b) Using graphing we find a point to consider.

$f(1.41) = 4.970728$

$f(1.42) = 5.020545$

Since $f(1.41) < 5 < f(1.42)$

by IVT there is a c in $(1.41, 1.42)$

s.t. $f(c) = 5$.

Other possible intervals: $(-5.01, -5)$

$(5.61, 5.62)$

10. (10pts) Consider the limit $\lim_{x \to 1} \frac{\log x}{x - 1}$. Use your calculator (careful with entering the denominator!) to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\frac{\log x}{x-1}$
1.1	0.413927
1.01	0.413214
1.001	0.434077
1.0001	0.434273
1.00001	0.434292
1.00001	0.434294

x	$\frac{\log x}{x-1}$
0.9	0.457574
0.99	0.436481
0.999	0.434512
0.9999	0.434316
0.99999	0.434296
0.999999	0.434294

It appears that $\lim_{x \to 1} \frac{\log x}{x-1} = 0.434294$

11. (12pts) Consider the function defined below. Find a value for c that makes the function continuous.

$$f(x) = \begin{cases} x^2 + \frac{cx}{16}, & \text{if } x \leq 4 \\ \frac{cx - 4c}{x^2 - 16}, & \text{if } x > 4. \end{cases}$$

For continuity, we must have:

$$\lim_{x \to 4-} f(x) = \lim_{x \to 4+} f(x)$$

$$\lim_{x \to 4-}\left(x^2 + \frac{cx}{16}\right) = \lim_{x \to 4+} \frac{c(x-4)}{x^2-16}$$

$$16 + \frac{c \cdot 4}{16} = \lim_{x \to 4+} \frac{c\cancel{(x-4)}}{\cancel{(x-4)}(x+4)}$$

$$16 + \frac{c}{4} = \frac{c}{4+4}$$

$$c = -8 \cdot 16 = -128$$

$$16 = \frac{c}{8} - \frac{c}{4}$$

$$16 = -\frac{c}{8}$$

Bonus. (10pts) Find the limit algebraically.

$$\lim_{x \to \infty} (\sqrt{x^2 + 5x + 2} - x) = \lim_{x \to \infty} \left(\sqrt{x^2 + 5x + 2} - x\right) \cdot \frac{\sqrt{x^2 + 5x + 2} + x}{\sqrt{x^2 + 5x + 2} + x}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2 + 5x + 2}^{\,2} - x^2}{\sqrt{x^2 + 5x + 2} + x} = \lim_{x \to \infty} \frac{x^2 + 5x + 2 - x^2}{\sqrt{x^2 + 5x + 2} + x}$$

factor out largest powers

$$= \lim_{x \to \infty} \frac{5x + 2}{\underbrace{\sqrt{x^2\left(1 + \frac{5}{x} + \frac{2}{x^2}\right)}}_{x\sqrt{1 + \frac{5}{x} + \frac{2}{x^2}}} + x} = \lim_{x \to \infty} \frac{\cancel{x}\left(5 + \underbrace{\frac{2}{x}}_{\to 0}\right)}{\cancel{x}\left(\sqrt{1 + \underbrace{\frac{5}{x}}_{\to 0} + \underbrace{\frac{2}{x^2}}_{\to 0}} + 1\right)}$$

$$= \frac{5 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{5}{2}$$