

Trigonometry — Joysheet 1
MAT 145, Spring 2017 — D. Ivanšić

Name: _____

Covers: 6.1, 6.2 Show all your work!

1. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\sin \theta = \frac{2}{9}$.

2. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\tan \theta = w$, where w is some number.

3. (10pts) Given that $\sin 23^\circ = a$, $\cos 34^\circ = b$, $\sec 15^\circ = c$ and $\cot 89^\circ = d$, use basic and cofunction identities to express the following quantities using a , b , c and d .

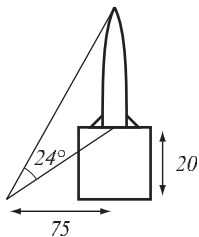
$$\sin 56^\circ = \qquad \qquad \qquad \csc 56^\circ = \qquad \qquad \qquad \tan 1^\circ = \qquad \qquad \qquad \tan 89^\circ =$$

$$\csc 75^\circ = \qquad \qquad \qquad \cos 15^\circ = \qquad \qquad \qquad \csc 23^\circ = \qquad \qquad \qquad \sec 67^\circ =$$

4. (10pts) Solve the right triangle (that is, find all sides and angles), if $a = 3$, $c = 7$.

5. (10pts) You are standing 45ft from a building and measure the angle of elevation to the top of the building to be 76° . How tall is the building?

6. (14pts) From a point on the ground 75 meters away from the launch pad, you observe a rocket and note it subtends an angle of 24° . If the launch pad is 20 meters tall, how tall is the rocket?



Trigonometry — Joysheet 2
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Name: _____
Covers: 6.3, 6.4 Show all your work!

1. (9pts) If $\cos \theta = -\frac{3}{7}$ and θ is in the third quadrant, find the exact values of all the trigonometric functions of θ . Draw a picture.

2. (7pts) The terminal side of angle θ is in the fourth quadrant and lies on the line $2x + 3y = 0$. Find the exact values of $\sin \theta$ and $\cot \theta$. Draw a picture.

3. (8pts) Sketch angles in standard position with indicated radian measure.

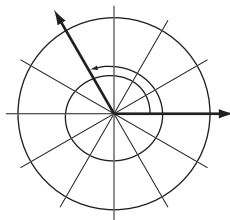
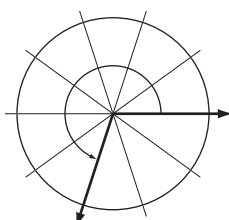
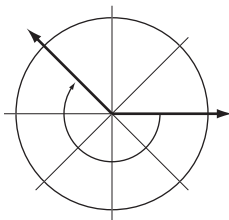
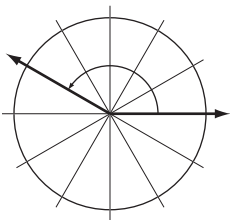
$$\frac{7\pi}{6}$$

$$\frac{3\pi}{7}$$

$$-\frac{4\pi}{3}$$

$$\frac{34\pi}{5}$$

4. (8pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced lines to help you determine what the angles are.)



5. (8pts) Chicago, IL, is directly north of Mobile, AL and their latitudes are $41^{\circ}50'13''\text{N}$ and $30^{\circ}41'40''\text{N}$, respectively. What is the distance along the Earth's surface between the cities, if the radius of Earth is 3960 miles?

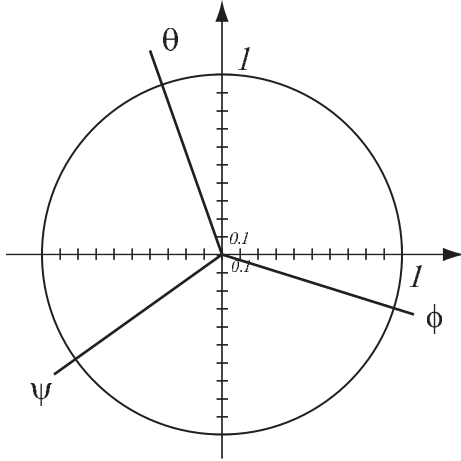
6. (8pts) The tip of the second-hand on a clock is 5 centimeters away from the center. As the second-hand rotates, what is its linear speed in centimeters per second?

7. (12pts) A truck whose tires have outside diameter 50in is traveling at 45mph.
a) What is the angular speed of the tires?
b) How many revolutions per minute do the tires make?

Trigonometry — Joysheet 3
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Name: _____
Covers: 6.5, 6.6 Show all your work!

1. (12pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn.



$$\cos \theta = \qquad \qquad \tan \theta =$$

$$\sin \psi = \qquad \qquad \cot \psi =$$

$$\sin \phi = \qquad \qquad \sec \phi =$$

2. (18pts) For each of the following, draw the unit circle and the appropriate angle in order to infer from the picture the exact values of the trigonometric functions.

$$\sin 210^\circ = \qquad \qquad \tan \frac{2\pi}{3} = \qquad \qquad \csc(-90^\circ) =$$

$$\cot(-150^\circ) = \qquad \qquad \sec\left(-\frac{5\pi}{6}\right) = \qquad \qquad \cos \frac{5\pi}{3} =$$

$$\sec 495^\circ = \qquad \qquad \tan\left(-\frac{9\pi}{2}\right) = \qquad \qquad \sin \frac{47\pi}{3} =$$

3. (10pts) Draw two periods of the graph of $y = 2 \sin(4x)$.

What is the amplitude? The period?

For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

4. (10pts) Draw two periods of the graph of $y = -3 \cos\left(2x + \frac{\pi}{3}\right)$.

What is the amplitude? The period?

For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

5. (10pts) Draw two periods of the graph of $y = \sin\left(-2x + \frac{\pi}{2}\right) + 1$.

What is the amplitude? The period?

For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

Trigonometry — Joysheet 4
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Name: _____

Covers: 7.1, 7.2 Show all your work!

1. (10pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\cos \alpha = -\frac{2}{5}$ and $\sin \beta = \frac{3}{7}$. Find the exact value of $\cos(\alpha + \beta)$.

2. (4pts) Use an identity to find the exact value of the expression (do not use the calculator):

$$\sin 93^\circ \cos 33^\circ - \cos 93^\circ \sin 33^\circ =$$

3. (8pts) Find the exact value of $\cos 112.5^\circ$ (do not use the calculator).

4. (10pts) Use identities to simplify the following expressions.

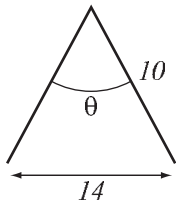
$$\sin\left(\frac{\pi}{2} - \theta\right) \cos \theta - \sin(-\theta) \cos\left(\frac{\pi}{2} - \theta\right) =$$

$$\frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\cos \theta} \cdot \frac{\sin \theta}{\cos(-\theta)} + \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos \theta} =$$

5. (8pts) Show the identity.

$$\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta$$

6. (10pts) A 10-ft folding ladder is placed on a floor so that its ends are 14 feet apart. Find the exact value for $\sin \theta$ (do not use the calculator), where θ is the angle the ladder subtends.



7. (10pts) Develop the formula for $\sin(3\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin \theta$ and $\cos \theta$ in it.

$$\sin(3\theta) = \sin(2\theta + \theta) =$$

Trigonometry — Joysheet 5
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Name: _____

Covers: 7.4, 7.5 Show all your work!

1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$$\arccos \frac{\sqrt{3}}{2} = \quad \arcsin \left(-\frac{\sqrt{2}}{2} \right) = \quad \arctan(-\sqrt{3}) = \quad \arccos(-2) =$$

2. (7pts) Find the exact value of the expressions (do not use the calculator). For some of them, you will need a picture.

$$\sin(\arcsin(0.83)) = \quad \arccos \left(\cos \frac{2\pi}{7} \right) = \quad \arcsin \left(\sin \frac{7\pi}{5} \right) =$$

3. (5pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\tan \left(\arccos \left(-\frac{1}{3} \right) \right) =$$

4. (5pts) Solve the equation (give a general formula for all solutions).

$$2 \sin \theta - \sqrt{2} = 0$$

5. (5pts) Use your calculator to solve the equation on the interval $[0^\circ, 360^\circ)$ (answers in degrees). A picture will help.

$$\sin \theta = -0.3$$

6. (10pts) Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0, 2\pi)$.

$$2 \cos^2 \theta - 5 \cos \theta - 3 = 0$$

7. (7pts) Solve the equation on the interval $[0, 2\pi)$.

$$\sin(2\theta) + 2 \sin^2 \theta = 0$$

8. (7pts) Solve the equation (give a general formula for all the solutions).

$$\sec^2 \theta = 6 \tan \theta + 8$$

9. (6pts) Find the exact value of the expression (do not use the calculator).

$$\sin \left(2 \arccos \left(-\frac{4}{7} \right) \right) =$$

Trigonometry — Joysheet 6
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Name: _____

Covers: 8.1, 8.2

Show all your work!

1. (4pts) Solve the triangle: $c = 4$, $C = 37^\circ$, $b = 8$.

2. (14pts) Solve the triangle: $b = 9$, $c = 11$, $B = 53^\circ$

3. (9pts) Solve the triangle: $a = 5$, $c = 2$, $B = 111^\circ$.

4. (10pts) Observing a building, you find that the angle of elevation to its top is 41° . Then you move toward the building 100 feet and get an angle of elevation of 66° to the top of the building. How tall is the building? (Draw a picture.)

5. (14pts) Naval rescue base B is located 35 kilometers $N68^\circ E$ from naval rescue base A . A ship in distress has position $S72^\circ E$ of base A and is 26 kilometers from base B . This allows for two possibilities for the ship's position (draw the picture). Assuming the position that is closer to base A , how far is the ship from base A ?

6. (9pts) A triangular plot of land with side lengths 101, 153 and 190 meters is being considered for purchase. What is its area?

Trigonometry — Joysheet 7
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Name: _____

Covers: 8.4

Show all your work!

1. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$$(r, \theta) = \left(3, \frac{7\pi}{6}\right)$$

$$(r, \theta) = \left(-2, \frac{3\pi}{4}\right)$$

2. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ . For each point, give three answers in polar coordinates, at least one of which has a negative r . Give exact answers — do not use the calculator.

$$(x, y) = (1, -2)$$

$$(x, y) = (-4, -4)$$

$$(x, y) = (-5, 2)$$

3. (6pts) Convert to a polar equation.

$$x^2 + 2xy - y^2 = 7$$

4. (6pts) Convert to a rectangular equation.

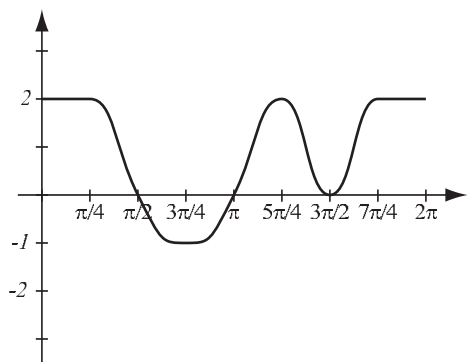
$$r = \cos(2\theta)$$

5. (12pts) Graph the equation $r = 4 \cos(3\theta)$ by doing the following:

a) Graph the equation in rectangular $r-\theta$ coordinates.

b) Use the information from a) to help you graph the equation in polar coordinates. Indicate corresponding parts of the graph in a) and b). Check your work with the graphing calculator.

6. (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular $r-\theta$ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates indicating corresponding parts of the graphs.



7. (8pts) Use your calculator to draw accurate graphs of the following polar curves.

$$r = 5 + 3 \sin(6\theta)$$

$$r = 3 + 5 \cos(3\theta)$$