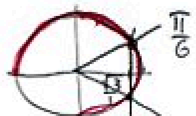
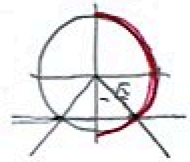
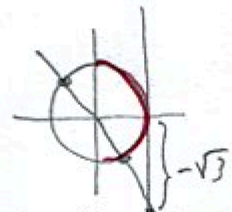
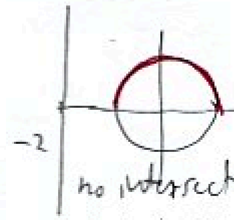
Trigonometry — Joysheet 5 **MAT 145, Spring 2017 — D. Ivanšić**	**Name:** Saul Ocean *Covers: 7.4, 7.5* *Show all your work!*

1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$ $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$ $\arccos(-2) =$ not defined



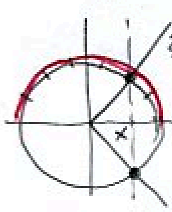





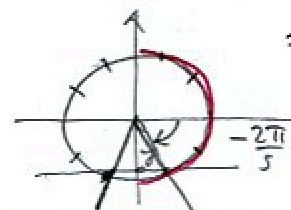
no intersection with unit circle

2. (7pts) Find the exact value of the expressions (do not use the calculator). For some of them, you will need a picture.

$\sin(\arcsin(0.83)) = 0.83$ $\arccos\left(\underbrace{\cos\frac{2\pi}{7}}_{x}\right) = \arccos x$ $\arcsin\left(\underbrace{\sin\frac{7\pi}{5}}_{=y}\right) = \arcsin y$



$= \frac{2\pi}{7}$



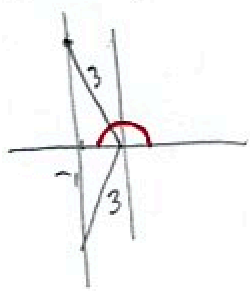
$= -\frac{2\pi}{5}$

3. (5pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$\tan\left(\underbrace{\arccos\left(-\frac{1}{3}\right)}_{=\theta}\right) = \tan\theta = -2\sqrt{2}$

$\cos\theta = -\frac{1}{3} = \frac{-1}{3} = \frac{x}{r}$

and θ in $[0, \pi]$



$(-1)^2 + y^2 = 3^2$

$y^2 = 8$

$y = \pm\sqrt{8}$

$y = \sqrt{8} = 2\sqrt{2}$

since θ is in $[0, \pi]$

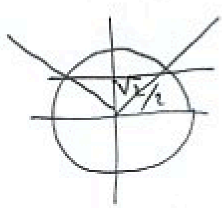
$\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$

4. (5pts) Solve the equation (give a general formula for all solutions).

$2\sin\theta - \sqrt{2} = 0$

$2\sin\theta = \sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$



$\theta = \frac{\pi}{4} + k \cdot 2\pi$

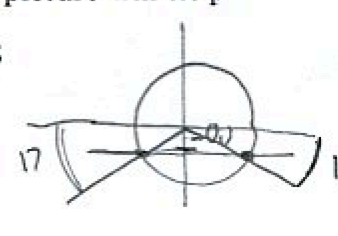
$\frac{3\pi}{4} + k \cdot 2\pi$

5. (5pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\sin\theta = -0.3$

$\arcsin(-0.3) = -17.457603 \leftarrow$ needs to be in $[0°, 360°)$

Need $360° - 17.4...° = 342.542397$



and $180° + 17.4° = 197.457603$

6. (10pts) Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta - 5\cos\theta - \cancel{1}\,3 = 0$

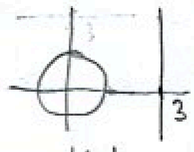
Let $u = \cos\theta$

$2u^2 - 5u - 3 = 0$

$u = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2}$

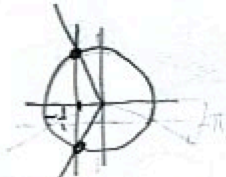
$= \dfrac{5 \pm \sqrt{25+24}}{4} = \dfrac{5 \pm \sqrt{49}}{4} = \dfrac{12}{4}, \dfrac{-2}{4} = 3, -\frac{1}{2}$

$\cos\theta = 3$



no soln

$\sin\theta = -\frac{1}{2}$



$\theta = \frac{2\pi}{3} + k \cdot 2\pi, \; -\frac{2\pi}{3} + k \cdot 2\pi$

sol. in $[0, 2\pi)$: $\frac{2\pi}{3}, \frac{4\pi}{3}$

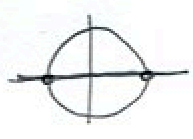
7. (7pts) Solve the equation on the interval $[0, 2\pi)$.

$\sin(2\theta) + 2\sin^2\theta = 0$

$2\sin\theta\cos\theta + 2\sin^2\theta = 0$

$2\sin\theta(\cos\theta + \sin\theta) = 0$

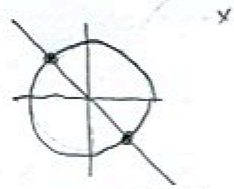
$\sin\theta = 0$ or $\cos\theta + \sin\theta = 0$



$\theta = 0, \pi$

$\sin\theta = -\cos\theta$

$y = -x$ on unit circle



$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

8. (7pts) Solve the equation (give a general formula for all the solutions).

$\sec^2\theta = 6\tan\theta + 8$

$\tan^2\theta + 1 = 6\tan\theta + 8$

$\tan^2\theta - 6\tan\theta - 7 = 0$

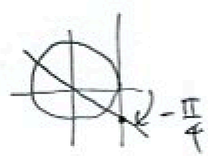
$u = \tan\theta$

$u^2 - 6u - 7 = 0$

$(u-7)(u+1) = 0$

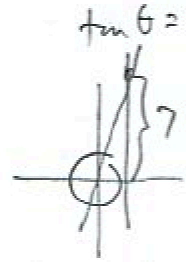
$u = -1, 7$

$\tan\theta = -1$



$\theta = -\frac{\pi}{4} + k \cdot \pi$

$\tan\theta = 7$



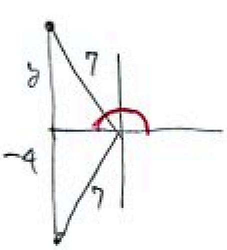
$\theta = \arctan 7 + k \cdot \pi$

9. (6pts) Find the exact value of the expression (do not use the calculator).

$\sin\left(\underbrace{2\arccos\left(-\frac{4}{7}\right)}_{\theta}\right) = \sin(2\theta) = 2\sin\theta\cos\theta = 2 \cdot \frac{\sqrt{33}}{7} \cdot \left(-\frac{4}{7}\right) = -\frac{8\sqrt{33}}{49}$

$\cos\theta = -\frac{4}{7} = \frac{-4}{7} = \frac{x}{r}$

and θ in $[0, \pi)$



$(-4)^2 + y^2 = 7^2$

$16 + y^2 = 49$

$y^2 = 33$

$y = \pm\sqrt{33} = \sqrt{33}$ since θ is in $[0, \pi]$

$\sin\theta = \frac{\sqrt{29}}{7}$