Trigonometry — Joysheet 4
MAT 145, Spring 2017 — D. Ivanšić

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Covers: 7.1, 7.2 *Show all your work!*

1. (10pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\cos\alpha = -\frac{2}{5}$ and $\sin\beta = \frac{3}{7}$. Find the exact value of $\cos(\alpha+\beta)$.

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= -\frac{2}{5}\cdot\left(\frac{-2\sqrt{10}}{7}\right) - \left(-\frac{\sqrt{21}}{5}\right)\cdot\frac{3}{7}$$
$$= \frac{4\sqrt{10}+3\sqrt{21}}{35}$$

$\cos\alpha = -\frac{2}{5} = \frac{-2}{5} = \frac{x}{r}$

$(-2)^2 + y^2 = 5^2$

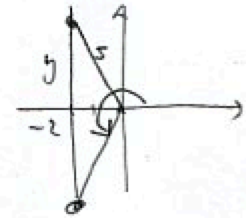
$4 + y^2 = 25$

$y^2 = 21$

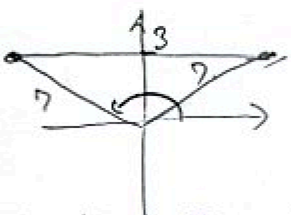
$y = \pm\sqrt{21}$

$y = -\sqrt{21}$ due to $\pi < \alpha < \frac{3\pi}{2}$

$\sin\alpha = -\frac{\sqrt{21}}{5}$



$\sin\beta = \frac{3}{7} = \frac{y}{r}$



$x^2 + 3^2 = 7^2$

$x^2 + 9 = 49$

$x^2 = 40$

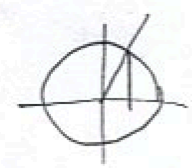
$x = \pm\sqrt{40} = \pm 2\sqrt{10}$

$x = -2\sqrt{10}$ due to $\frac{\pi}{2} < \beta < \pi$

$\cos\beta = -\frac{2\sqrt{10}}{7}$

2. (4pts) Use and identity to find the exact value of the expression (do not use the calculator):

$\sin 93^\circ \cos 33^\circ - \cos 93^\circ \sin 33^\circ = \sin(93^\circ - 33^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$



3. (8pts) Find the exact value of $\cos 112.5^\circ$ (do not use the calculator).

$112.5^\circ = \frac{225^\circ}{2}$

$\cos^2\frac{225^\circ}{2} = \frac{1+\cos 225^\circ}{2}$

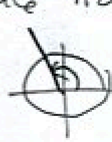
$\cos^2 112.5^\circ = \frac{1-\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2}$

$= \frac{2-\sqrt{2}}{4}$

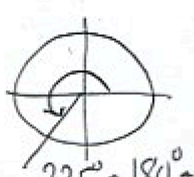
$\cos 112.5^\circ = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$

$\cos 112.5^\circ = -\frac{\sqrt{2-\sqrt{2}}}{2}$

since 112.5° is in quad. 2



$225^\circ = 180^\circ + 45^\circ$



4. (10pts) Use identities to simplify the following expressions.

$\sin\left(\frac{\pi}{2}-\theta\right)\cos\theta - \sin(-\theta)\cos\left(\frac{\pi}{2}-\theta\right) = \cos\theta\cdot\cos\theta - (-\sin\theta)\sin\theta = \cos^2\theta + \sin^2\theta = 1$

$\dfrac{\cos\left(\frac{\pi}{2}-\theta\right)}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos(-\theta)} + \dfrac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\theta} = \underbrace{\dfrac{\sin\theta}{\cos\theta}}_{\tan\theta}\cdot\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} = \tan^2\theta + 1 = \sec^2\theta$

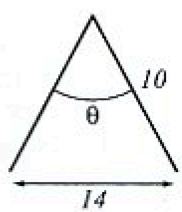
5. (8pts) Show the identity.

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$$

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=\frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}=\frac{2}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=$$

$$=2\cdot\frac{1}{\cos^2\theta}=2\sec^2\theta$$

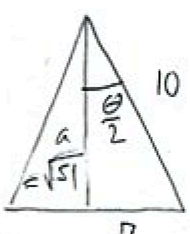
6. (10pts) A 10-ft folding ladder is placed on a floor so that its ends are 14 feet apart. Find the exact value for $\sin\theta$ (do not use the calculator), where θ is the angle the ladder subtends.



$$\sin\theta=\sin\left(2\cdot\frac{\theta}{2}\right)$$

$$=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$=\overset{1}{\cancel{2}}\cdot\frac{7}{\underset{5}{\cancel{10}}}\cdot\frac{\sqrt{51}}{10}=\frac{7\sqrt{51}}{50}$$



$\sin\frac{\theta}{2}=\frac{7}{10}$

$\cos\frac{\theta}{2}=\frac{\sqrt{51}}{10}$

$a^2+7^2=10^2$

$a^2=100-49$

$a^2=51,\quad a=\sqrt{51}$

7. (10pts) Develop the formula for $\sin(3\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$$\sin(3\theta)=\sin(2\theta+\theta)=\sin(2\theta)\cos\theta+\cos(2\theta)\sin\theta$$

$$=2\sin\theta\cos\theta\cos\theta+(\cos^2\theta-\sin^2\theta)\sin\theta$$

$$=2\sin\theta\cos^2\theta+\sin\theta\cos^2\theta-\sin^3\theta=\boxed{3\sin\theta\cos^2\theta-\sin^3\theta}$$

depending on which formula for $\cos 2\theta$ is used.

OR

$$=2\sin\theta\cos^2\theta+(2\cos^2\theta-1)\sin\theta$$

$$=2\sin\theta\cos^2\theta+2\sin\theta\cos^2\theta-\sin\theta=\boxed{4\sin\theta\cos^2\theta-\sin\theta}$$

OR

$$=2\sin\theta\cos^2\theta+(1-2\sin^2\theta)\sin\theta=\boxed{2\sin\theta\cos^2\theta-2\sin^3\theta+\sin\theta}$$

$=$