Trigonometry — Exam 1
MAT 145, Spring 2017— D. Ivanšić

Name: _____

Show all your work!

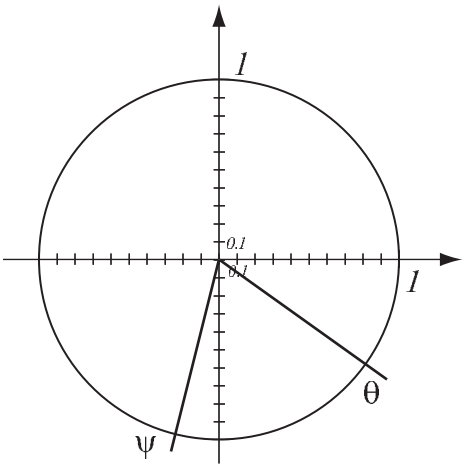
1. (10pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\tan\theta = \frac{1}{5}$. Draw a picture.

2. (12pts) If $\cos\theta = -\frac{2}{5}$ and θ is in the third quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

3. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\sin 150^\circ =$ $\cos\dfrac{5\pi}{4} =$ $\csc(-180^\circ) =$ $\tan\dfrac{10\pi}{3} =$

4. (9pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn. Note the angles are **not** the standard angles.



$\sin\theta =$ $\sec\theta =$

$\cos\psi =$ $\tan\psi =$

5. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$20^\circ =$

$\frac{13\pi}{12}$ radians $=$

6. (6pts) Use your calculator to evaluate (round to 6 decimals):

$\tan 49^\circ =$ $\sec\frac{2\pi}{7} =$

7. (3pts) Use your calculator to find the acute angle θ (in degrees, round to 6 decimals) if

$\sin\theta = \frac{4}{17}$

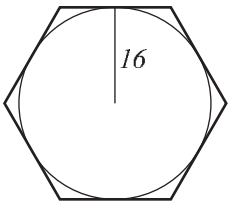
8. (10pts) Draw two periods of the graph of $y = 2\sin(4x + \pi)$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

9. (10pts) A kite attached to a 110 ft string is flying so that the angle of elevation from the ground anchor to the kite is $35°$. How high above the ground is the kite?

10. (10pts) Apple's new headquarters building is in the shape of a ring with outer diameter 460 meters. If we refer to points on the circle via correspondence to a clock, how far would a person have to walk along the outside wall to get from a point at 1 o'clock to a point at 6 o'clock?

11. (12pts) The Earth rotates around the sun on an approximately circular path of radius 91.4 million miles. It takes the Earth 365.25 days for one complete revolution (hence the leap years!).
a) What is Earth's angular velocity due to this rotation in radians per hour?
b) What is Earth's linear velocity due to this rotation in miles per hour?

Bonus. (10pts) A circle of radius 16 meters is inscribed in a regular hexagon. Find the exact value of the perimeter of the hexagon (not a calculator approximation).



Trigonometry — Exam 2
MAT 145, Spring 2017— D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (16pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\sin 75^\circ =$

$\tan 157.5^\circ =$

2. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin \frac{1}{2} =$ $\qquad \arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\qquad \arcsin(4) =$ $\qquad \arctan \frac{1}{\sqrt{3}} =$

3. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\sin(\arcsin(-0.4)) =$ $\qquad \arccos\left(\cos \frac{9\pi}{7}\right) =$

4. (7pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\cos\left(\arctan\left(-\frac{7}{4}\right)\right) =$$

5. (8pts) Use identities to simplify the following expression.

$$\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\theta}+\cos\left(\frac{\pi}{2}-\theta\right)\sin(-\theta) =$$

Show the identities:

6. (8pts) $\tan\theta(\tan\theta+\cot\theta)=\sec^2\theta$

7. (8pts) $(\sin\theta+\cos\theta)^2=1+\sin(2\theta)$

8. (5pts) Solve the equation in radians (give a general formula for all solutions).

$2\cos\theta + \sqrt{3} = 0$

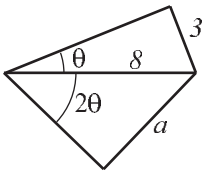
9. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.8$

10. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta + \cos\theta - 1 = 0$

11. (12pts) The two triangles in the picture are right triangles. One of them has an angle of measure θ, the other, 2θ. Find the exact value for the length of side a (do not use the calculator).



Bonus. (10pts) Develop the formula for $\cos(4\theta)$ by using sum or double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

Trigonometry — Exam 3
MAT 145, Spring 2017— D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (6pts) Solve the triangle: $a = 8$, $b = 3$, $c = 4$.

2. (14pts) Solve the triangle: $b = 10$, $c = 7$, $B = 44^\circ$

3. (13pts) Solve the triangle: $b = 3$, $c = 2$, $A = 79^\circ$.

4. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(2, \frac{5\pi}{6}\right)$ $(r, \theta) = \left(-5, -\frac{3\pi}{4}\right)$

5. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x, y) = (3, -3)$ $(x, y) = (-2\sqrt{3}, 2)$

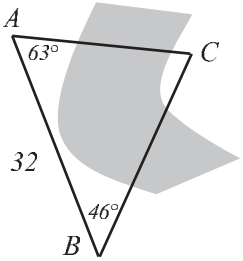
6. (9pts) Convert to a polar equation. Answer should be solved for r.

$x^2 + 2xy + y^2 = 5$

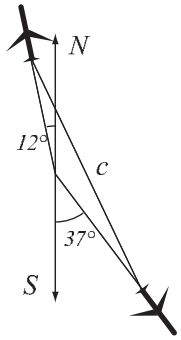
7. (8pts) The vertices of a triangle are given in **polar coordinates**: $A = (0, 0)$, $B = \left(4, \frac{\pi}{3}\right)$, $C = \left(5, \frac{\pi}{2}\right)$.
a) Draw the triangle.
b) Find the exact area of the triangle (do not use the calculator).

8. (8pts) Use your calculator to draw an accurate graph of the polar curve $r = 1 + 5\cos(4\theta)$.

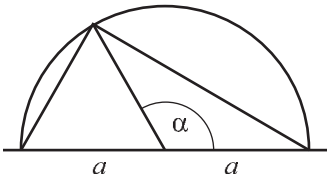
9. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is 63° and the angle between lines of sight BA and BC from point B is 46°.
a) How far apart are A and C?
b) How far apart are B and C?



10. (13pts) Two planes leave an airport: one flies $N12°W$ at 450 mph, and the other flies $S37°E$ at 540 mph. What is the distance c between the planes after two hours?



Bonus. (10pts) In a circle of radius a, the large triangle, whose bottom side is a diameter, is split into two triangles as shown.
a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.



Trigonometry — Final Exam
MAT 145, Spring 2017— D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (12pts) If $\sin\theta = -\frac{5}{8}$ and θ is in the fourth quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

2. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\cos 60^\circ =$ $\qquad$ $\sin \frac{5\pi}{4} =$ $\qquad$ $\sec(-90^\circ) =$ $\qquad$ $\tan \frac{8\pi}{3} =$

3. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos \frac{\sqrt{3}}{2} =$ $\qquad$ $\arcsin\left(-\frac{\sqrt{2}}{2}\right) =$ $\qquad$ $\arccos(2) =$ $\qquad$ $\arctan(-\sqrt{3}) =$

4. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\sin(\arcsin 0.2) =$ $\qquad\qquad$ $\arccos\left(\cos\dfrac{8\pi}{5}\right) =$

5. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$63^\circ =$

$\dfrac{7\pi}{15}$ radians $=$

6. (10pts) Apple's new headquarters building is in the shape of a ring with outer diameter 460 meters. If we refer to points on the circle via correspondence to a clock, how far would a person have to walk along the outside wall to get from a point at 1 o'clock to a point at 8 o'clock, going the long way?

7. (8pts) Use an identity (sum, difference, half- or double-angle) to find the exact value of the trigonometric function below (do not use the calculator).

$\cos 195^\circ =$

8. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.25$

9. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\sin^2\theta - \sin\theta - 1 = 0$

10. (14pts) Solve the triangle: $a = 7$, $c = 6$, $A = 38°$

11. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(3, \frac{\pi}{6}\right)$ $(r, \theta) = \left(-4, \frac{3\pi}{4}\right)$

12. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x, y) = (5\sqrt{3}, -5)$ $(x, y) = (-4, 4)$

13. (10pts) A kite attached to a 60 ft string is flying so that the angle of elevation from the ground anchor to the kite is 49°. How high above the ground is the kite?

14. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is $63°$ and the angle between lines of sight BA and BC from point B is $46°$.
a) How far apart are A and C?
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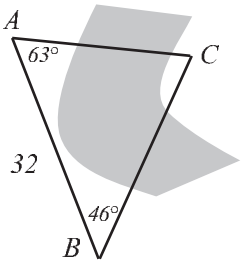
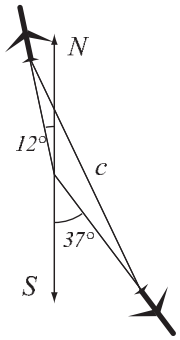
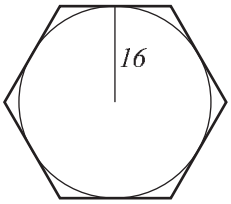


15. (13pts) Two planes leave an airport: one flies $N12°W$ at 250 mph, and the other flies $S37°E$ at 300 mph. What is the distance c between the planes after two hours?



Bonus. (7pts) A circle of radius 16 meters is inscribed in a regular hexagon. Find the exact value of the perimeter of the hexagon (not a calculator approximation).



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a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.