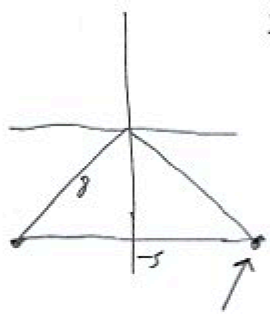
Trigonometry — Final Exam **MAT 145, Spring 2017— D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (12pts) If $\sin\theta = -\frac{5}{8}$ and θ is in the fourth quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.



$\sin\theta = \frac{-5}{8} = \frac{y}{r}$

$x^2 + (-5)^2 = 8^2$

$x^2 + 25 = 64$

$x^2 = 39$

$x = \pm\sqrt{39}$

$x = \sqrt{39}$ due to fourth quad.

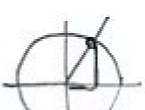
$\sin\theta = -\frac{5}{8}$ $\quad \csc\theta = -\frac{8}{5}$

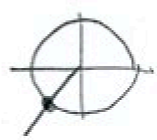
$\cos\theta = \frac{\sqrt{39}}{8}$ $\quad \sec\theta = \frac{8}{\sqrt{39}}$

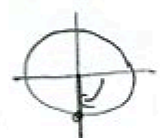
$\tan\theta = -\frac{5}{\sqrt{39}}$ $\quad \cot\theta = -\frac{\sqrt{39}}{5}$

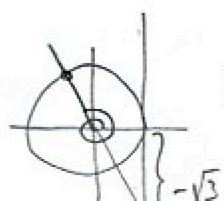
2. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\cos 60° = \frac{1}{2}$ $\qquad \sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ $\qquad \sec(-90°) = \frac{1}{\cos(-90°)} = \frac{1}{0}$. not defined $\qquad \tan\frac{8\pi}{3} = -\sqrt{3}$





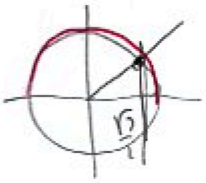


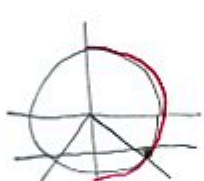


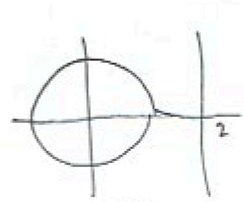
$\frac{8\pi}{3} = 2\frac{2}{3}\pi$

3. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

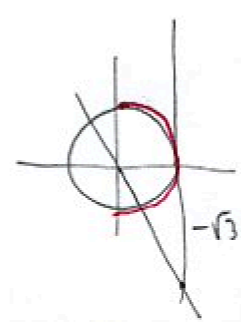
$\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ $\qquad \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ $\qquad \arccos(2) =$ not defined $\qquad \arctan(-\sqrt{3}) = -\frac{\pi}{3}$







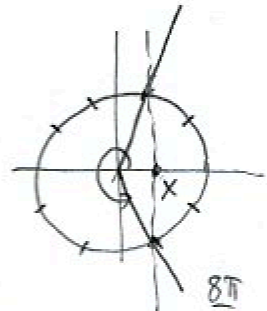
no intersection



4. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\sin(\arcsin 0.2) =$ 0.2

$\arccos\left(\cos \frac{8\pi}{5}\right) =$ arccos x $= \frac{2\pi}{5}$

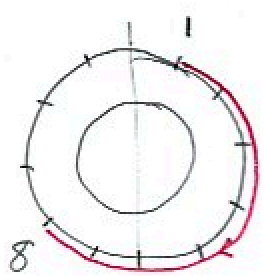


5. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$63° = \frac{\cancel{63}^{7}°\cdot\pi}{\cancel{180°}_{20}} = \frac{7\pi}{20} = 1.099557$

$\frac{7\pi}{15}$ radians $= \frac{7\cancel{\pi}}{\cancel{15}_{1}} \cdot \frac{\cancel{180°}^{12}}{\cancel{\pi}} = 84°$

6. (10pts) Apple's new headquarters building is in the shape of a ring with outer diameter 460 meters. If we refer to points on the circle via correspondence to a clock, how far would a person have to walk along the outside wall to get from a point at 1 o'clock to a point at 8 o'clock, going the long way?



$s = r\theta = 230 \cdot \frac{7\pi}{6} = 842.994029$ meters

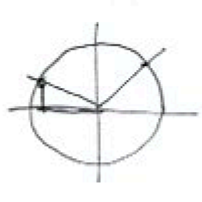
$\theta = 7 \cdot \frac{\pi}{6} = \frac{7\pi}{6}$ $\quad r = \frac{460}{2} = 230$

(every "hour" is angle $\frac{\pi}{6}$)

7. (8pts) Use an identity (sum, difference, half- or double-angle) to find the exact value of the trigonometric function below (do not use the calculator).

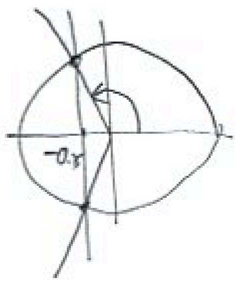
$\cos 195° = \cos(150° + 45°) = \cos 150° \cos 45° - \sin 150° \sin 45°$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{6}-\sqrt{2}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}$



8. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.25$



$\theta = \arccos(-0.25) = 104.477512°$

or

$\theta = 360 - \arccos(-0.25) = 255.522488°$

9. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\sin^2\theta - \sin\theta - 1 = 0$

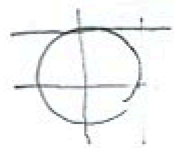
Let $u = \sin\theta$

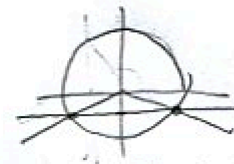
$2u^2 - u - 1 = 0$

$u = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$

$= \dfrac{1 \pm \sqrt{9}}{4} = \dfrac{1 \pm 3}{4} = 1, -\frac{1}{2}$

$\sin\theta = 1$ or $\sin\theta = -\frac{1}{2}$





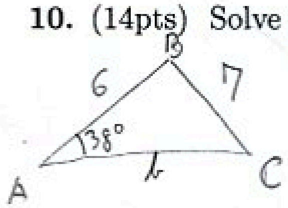
a) $\theta = \frac{\pi}{2} + k \cdot 2\pi$

b) $\theta = \frac{\pi}{2}$

a) $\theta = -\frac{\pi}{6} + k \cdot 2\pi, \ -\frac{5\pi}{6} + k \cdot 2\pi$

b) $\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}$

10. (14pts) Solve the triangle: $a = 7$, $c = 6$, $A = 38°$



$\dfrac{6}{\sin C} = \dfrac{7}{\sin 38°}$

$6\sin 38° = 7\sin C$

$\sin C = \dfrac{6\sin 38°}{7} = 0.52771$

$C = \arcsin 0.52\ldots$

$= 31.850848°$

or

$C = 180° - \arcsin 0.52\ldots$

$= 148.149152°$

Since $148.14 + 38° > 180°$

this is not a solution

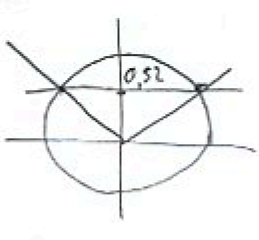
$B = 180° - (38° + 31.85°\ldots)$

$= 110.149152°$

$\dfrac{b}{\sin 110.14\ldots} = \dfrac{7}{\sin 38°}$

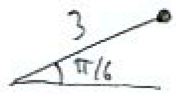
$b = \dfrac{7}{\sin 38°} \cdot \sin(110.14\ldots)$

$= 10.674037$



11. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

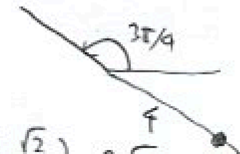
$(r, \theta) = \left(3, \frac{\pi}{6}\right)$



$x = 3\cos\frac{\pi}{6} = 3\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$y = 3\sin\frac{\pi}{6} = 3\cdot\frac{1}{2} = \frac{3}{2}$

$(r, \theta) = \left(-4, \frac{3\pi}{4}\right)$



$x = -4\cos\frac{3\pi}{4} = -4\left(-\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

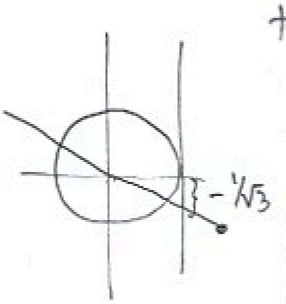
$y = -4\sin\frac{3\pi}{4} = -4\cdot\frac{\sqrt{2}}{2} = -2\sqrt{2}$

12. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x, y) = (5\sqrt{3}, -5)$

$r = \sqrt{(5\sqrt{3})^2 + (-5)^2} = \sqrt{25\cdot 3 + 25} = \sqrt{100} = 10$

$\tan\theta = \frac{-5}{5\sqrt{3}} = -\frac{1}{\sqrt{3}}$



$\theta = -\frac{\pi}{6}$

$\left(10, -\frac{\pi}{6}\right)$

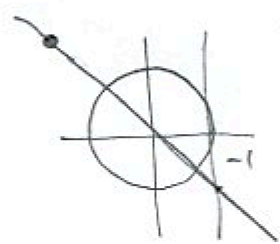
$\left(10, \frac{11\pi}{6}\right)$

$\left(-10, \frac{5\pi}{6}\right)$

$(x, y) = (-4, 4)$

$r = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = \frac{4}{-4} = -1$



$\theta = \frac{3\pi}{4}$

$\left(4\sqrt{2}, \frac{3\pi}{4}\right)$

$\left(4\sqrt{2}, -\frac{5\pi}{4}\right)$

$\left(-4\sqrt{2}, -\frac{\pi}{4}\right)$

13. (10pts) A kite attached to a 60 ft string is flying so that the angle of elevation from the ground anchor to the kite is 49°. How high above the ground is the kite?

$\frac{h}{60} = \sin 49^\circ$

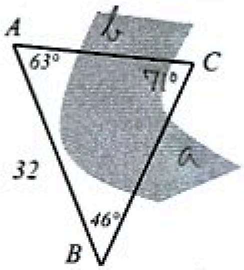
$h = 60\sin 49^\circ$

$= 45.282575\,\text{m}$

14. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is 63° and the angle between lines of sight BA and BC from point B is 46°.

a) How far apart are A and C?

b) How far apart are B and C?



$C = 180° - (63° + 46°)$

$= 71°$

a) $\dfrac{b}{\sin 46°} = \dfrac{32}{\sin 71°}$

$b = \dfrac{32 \sin 46°}{\sin 71°}$

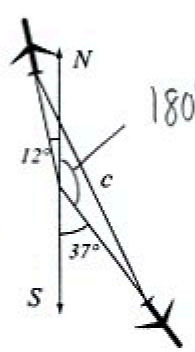
$= 24.345237 \text{ m}$

b) $\dfrac{a}{\sin 63°} = \dfrac{32}{\sin 71°}$

$a = \dfrac{32 \sin 63°}{\sin 71°}$

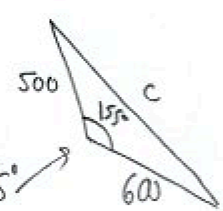
$= 30.155102 \text{ m}$

15. (13pts) Two planes leave an airport: one flies $N12°W$ at 250 mph, and the other flies $S37°E$ at 300 mph. What is the distance c between the planes after two hours?



$180° - 37° = 143°$

$143° + 12° = 155°$



$250 \cdot 2 = 500$

$300 \cdot 2 = 600$

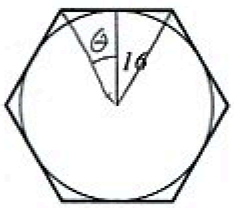
$c^2 = 500^2 + 600^2 - 2 \cdot 500 \cdot 600 \cos 155°$

$c^2 = 250000 + 360000 - 600000 \cos 155°$

$c^2 = 1153784.672$

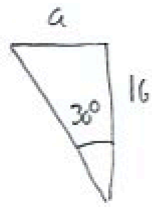
$c = 1074.143693$ miles

Bonus. (7pts) A circle of radius 16 meters is inscribed in a regular hexagon. Find the exact value of the perimeter of the hexagon (not a calculator approximation).



$\theta \cdot 12 = 360^\circ$

$\theta = 30^\circ$

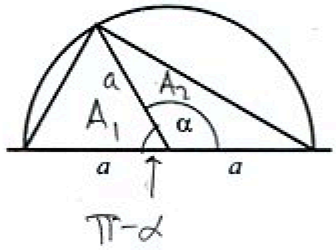


$\frac{a}{16} = \tan 30^\circ$

$a = 16 \tan 30^\circ = 16 \cdot \frac{1}{\sqrt{3}}$

$\text{Perimeter} = 12a = 12 \cdot \frac{16}{\sqrt{3}} = \frac{192}{\sqrt{3}} = \frac{\overset{64}{\cancel{192}}\sqrt{3}}{\underset{1}{\cancel{3}}} = 64\sqrt{3}$ meters

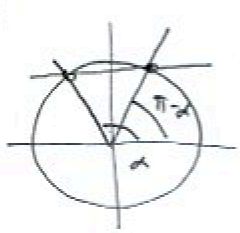
Bonus. (8pts) In a circle of radius a, the large triangle, whose bottom side is a diameter, is split into two triangles as shown.
a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.



a) $A_2 = \frac{1}{2} a \cdot a \cdot \sin\alpha = \frac{a^2 \sin\alpha}{2}$

$A_1 = \frac{1}{2} a \cdot a \cdot \sin(\pi - \alpha) = \frac{a^2 \sin(\pi-\alpha)}{2}$

b)



Since $\sin\alpha = \sin(\pi - \alpha)$, areas are equal.