Trigonometry — Exam 3
MAT 145, Spring 2017— D. Ivanšić

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Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (6pts) Solve the triangle: $a = 8$, $b = 3$, $c = 4$.

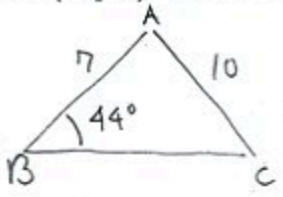
$3 + 4 < 8$

so not possible to build triagle

or: $\cos C = \frac{8^2 + 3^2 - 4^2}{2 \cdot 8 \cdot 3} = \frac{57}{48} = 1.1875 > 1$

so no solution

2. (14pts) Solve the triangle: $b = 10$, $c = 7$, $B = 44°$



$\frac{7}{\sin C} = \frac{10}{\sin 44°}$

$7 \sin 44° = 10 \sin C$

$\sin C = \frac{7 \sin 44°}{10} = 0.486261$

$C = 29.095114°$

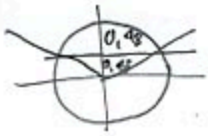
or $C = 180° - 29.05..$

$= 150.904886$ ← too big, since $44° + 150.90° > 180°$ this angle has no solution

$A = 180° - (44° + 29.09°) = 106.904886°$

$\frac{a}{\sin 106.90..} = \frac{10}{\sin 44°}$

$a = \frac{10 \sin 106.90°}{\sin 44°} = 13.773516$



3. (13pts) Solve the triangle: $b = 3$, $c = 2$, $A = 79°$.

$a^2 = 3^2 + 2^2 - 2 \cdot 3 \cdot 2 \cos 79°$

$a^2 = 13 - 12 \cos 79° = 10.71..$

$a = 3.272658$

$\cos B = \frac{3.27..^2 + 2^2 - 3^2}{2 \cdot 3.27.. \cdot 2} = 0.436212$

$B = 64.137555°$

$C = 180° - (79° + 64.13°)$

$= 36.862445$

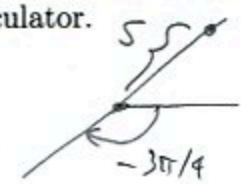
4. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r,\theta) = \left(2, \frac{5\pi}{6}\right)$

$x = 2\cos\frac{5\pi}{6} = 2\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$y = 2\sin\frac{5\pi}{6} = 2\cdot\frac{1}{2} = 1$

$(r,\theta) = \left(-5, -\frac{3\pi}{4}\right)$



$x = -5\cos\left(-\frac{3\pi}{4}\right) = -5\cdot\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$

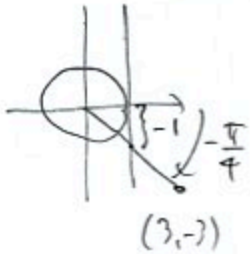
$y = -5\sin\left(-\frac{3\pi}{4}\right) = -5\cdot\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$

5. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x,y) = (3,-3)$

$r = \sqrt{3^2+(-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3} = -1, \quad \theta = -\frac{\pi}{4}$



$\left(3\sqrt{2}, -\frac{\pi}{4}\right)$

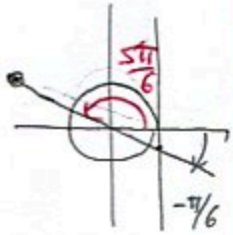
$\left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$\left(-3\sqrt{2}, \frac{3\pi}{4}\right)$

$(x,y) = (-2\sqrt{3}, 2)$

$r = \sqrt{(-2\sqrt{3})^2+2^2} = \sqrt{12+4} = 4$

$\tan\theta = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}, \quad \theta = \frac{5\pi}{6}$



$\left(4, \frac{5\pi}{6}\right)$

$\left(4, -\frac{7\pi}{6}\right)$

$\left(-4, -\frac{\pi}{6}\right)$

6. (9pts) Convert to a polar equation. Answer should be solved for r.

$x^2 + 2xy + y^2 = 5$

$(r\cos\theta)^2 + 2r\cos\theta\, r\sin\theta + (r\sin\theta)^2 = 5$

$r^2\cos^2\theta + 2r^2\sin\theta\cos\theta + r^2\sin^2\theta = 5$

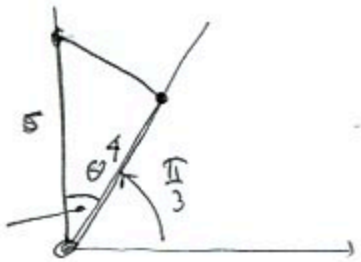
$r^2\underbrace{(\cos^2\theta+\sin^2\theta)}_{=1} + r^2\cdot\underbrace{2\sin\theta\cos\theta}_{=\sin(2\theta)} = 5$

$r^2(1+\sin(2\theta)) = 5$

$r^2 = \frac{5}{1+\sin(2\theta)}$

$r = \pm\sqrt{\frac{5}{1+\sin(2\theta)}}$

7. (8pts) The vertices of a triangle are given in **polar coordinates**: $A = (0, 0)$, $B = (4, \frac{\pi}{3})$, $C = (5, \frac{\pi}{2})$.
a) Draw the triangle.
b) Find the exact area of the triangle (do not use the calculator).

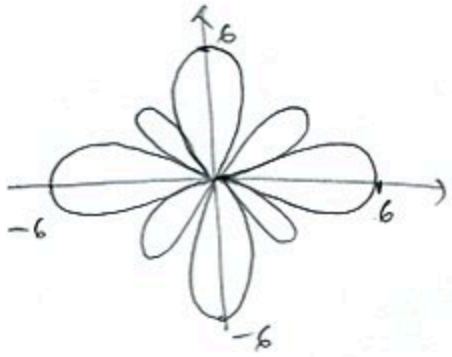


$\text{Area} = \frac{1}{2} 4 \cdot 5 \cdot \sin\theta = \frac{1}{2} \cdot 20 \cdot \sin\frac{\pi}{6} = 10 \cdot \frac{1}{2} = 5$

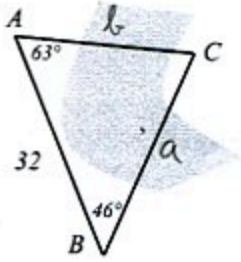
$\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \frac{\pi}{6}$

6

8. (8pts) Use your calculator to draw an accurate graph of the polar curve $r = 1 + 5\cos(4\theta)$.



9. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is 63° and the angle between lines of sight BA and BC from point B is 46°.
a) How far apart are A and C?
b) How far apart are B and C?



$C = 180° - (46° + 63°) = 71°$

a) $\frac{b}{\sin 46°} = \frac{32}{\sin 71°}$

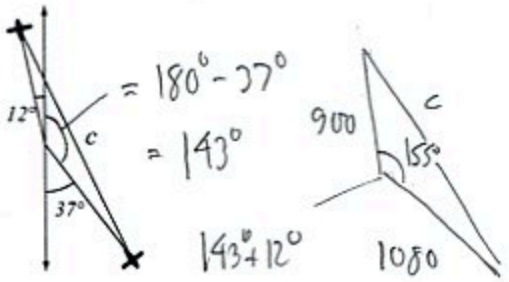
b) $\frac{a}{\sin 63°} = \frac{32}{\sin 71°}$

$b = \frac{32 \sin 46°}{\sin 71°} = 24.345237$ meters

$a = \frac{32 \sin 63°}{\sin 71°}$

$= 30.155102$

10. (13pts) Two planes leave an airport: one flies $N12°W$ at 450 mph, and the other flies $S37°E$ at 540 mph. What is the distance c between the planes after two hours?



$c^2 = 900^2 + 1080^2 - 2 \cdot 900 \cdot 1080 \cdot \cos 155°$

$= 3{,}738{,}262.338$

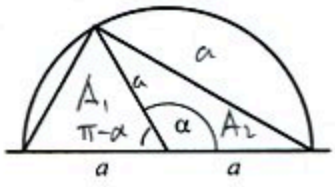
$c = 1933.458647$ miles

In 2 hours, the planes travel

$450 \cdot 2 = 900$ miles

$540 \cdot 2 = 1080$ miles

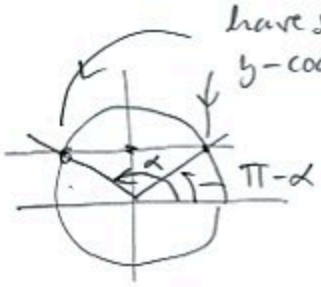
Bonus. (10pts) In a circle of radius a, the large triangle, whose bottom side is a diameter, is split into two triangles as shown.
a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.



a) $A_1 = \frac{1}{2} a \cdot a \cdot \sin(\pi - \alpha) = \frac{a^2}{2} \sin(\pi - \alpha)$

$A_2 = \frac{1}{2} a \cdot a \sin \alpha = \frac{a^2}{2} \sin \alpha$

b)



since $\sin(\pi - \alpha) = \sin \alpha$, area are equal.