Trigonometry — Exam 2 MAT 145, Spring 2017— D. Ivanšić	**Name:** Saul Ocean *Show all your work!*

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ $\qquad$ $\sin(2u) = 2 \sin u \cos u$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ $\qquad$ $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ $\qquad$ $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$

$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\qquad$ $\sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ $\qquad$ $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (16pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

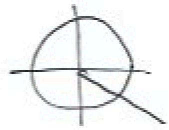
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\tan 157.5° =$

$\tan^2 \frac{315°}{2} = \frac{1 - \cos 315°}{1 + \cos 315°} = \frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} \cdot \frac{2}{2} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$

$157.5° = \frac{315°}{2}$

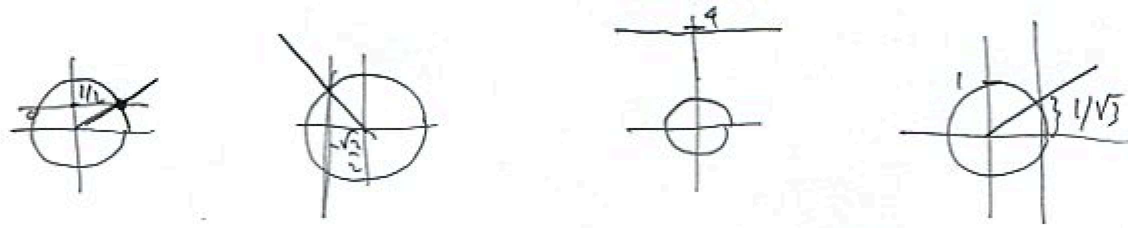
$315° = 270° + 45°$



$\tan 157.5° = \pm\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = -\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$ since 157.5° is in quad. 2

2. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

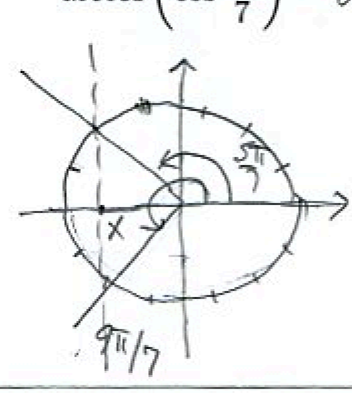
$\arcsin \frac{1}{2} = \frac{\pi}{6}$ $\qquad$ $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ $\qquad$ $\arcsin(4) =$ not def. $\qquad$ $\arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$



3. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\sin(\arcsin(-0.4)) = -0.4$ $\qquad$ $\arccos\left(\cos \frac{9\pi}{7}\right) = \arccos x = \frac{5\pi}{7}$

$2\pi - \frac{9\pi}{7}$, or from picture

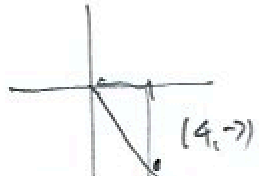


4. (7pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\cos\Big(\underbrace{\arctan\left(-\tfrac{7}{4}\right)}_{\theta}\Big) = \quad \cos\theta = \frac{x}{r} = \frac{4}{\sqrt{65}}$$

$\tan\theta = -\frac{7}{4} = \frac{-7}{4}$

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$r^2 = 4^2 + (-7)^2$

$r^2 = 16 + 49$

$r^2 = 65 \quad r = \sqrt{65}$

5. (8pts) Use identities to simplify the following expression.

$$\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos\theta} + \cos\left(\frac{\pi}{2} - \theta\right)\sin(-\theta) = \frac{\cos\theta}{\cos\theta} + \sin\theta \cdot (-\sin\theta) = 1 - \sin^2\theta = \cos^2\theta$$

Show the identities:

6. (8pts) $\tan\theta(\tan\theta + \cot\theta) = \sec^2\theta$

$$\tan\theta \cdot (\tan\theta + \cot\theta) = \tan^2\theta + \tan\theta \cdot \frac{1}{\tan\theta}$$

$$= \tan^2\theta + 1$$

$$= \sec^2\theta$$

7. (8pts) $(\sin\theta + \cos\theta)^2 = 1 + \sin(2\theta)$

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$

$$= \underbrace{\cos^2\theta + \sin^2\theta}_{=\,1} + \underbrace{2\sin\theta\cos\theta}_{\sin(2\theta)}$$

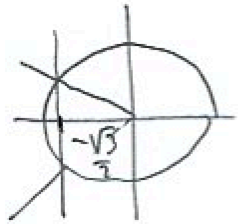
$$= 1 + \sin(2\theta)$$

8. (5pts) Solve the equation in radians (give a general formula for all solutions).

$2\cos\theta + \sqrt{3} = 0$

$2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

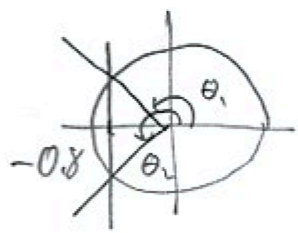


$\theta = \frac{5\pi}{6} + k \cdot 2\pi$

$\theta = -\frac{5\pi}{6} + k \cdot 2\pi$

9. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.8$



$\theta_1 = \arccos(-0.8) = 143.130102$

$\theta_2 = 360° - \theta_1 = 216.869898$

10. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta + \cos\theta - 1 = 0$

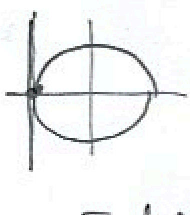
Let $u = \cos\theta$

$2u^2 + u - 1 = 0$

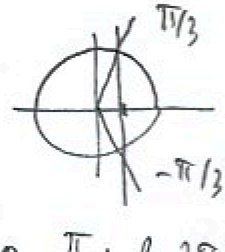
$u = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$

$= \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = -1, \frac{1}{2}$

$\cos\theta = -1$



$\cos\theta = \frac{1}{2}$



a) $\theta = \pi + k \cdot 2\pi$

$\theta = \frac{\pi}{3} + k \cdot 2\pi$

$\theta = -\frac{\pi}{3} + k \cdot 2\pi$

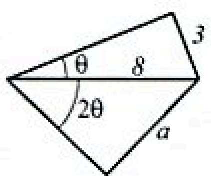
b) π

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\uparrow$

$2\pi - \frac{\pi}{3}$

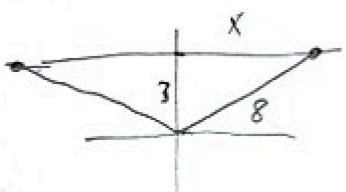
11. (12pts) The two triangles in the picture are right triangles. One of them has an angle of measure θ, the other, 2θ. Find the exact value for the length of side a (do not use the calculator).



$\sin\theta = \frac{3}{8}$ $\quad\quad$ $\sin(2\theta) = \frac{a}{8}$

Thus, $a = 8\sin(2\theta) = 8 \cdot 2\sin\theta\cos\theta$

$= \cancel{8}\cdot\cancel{2}\cdot\frac{3}{\cancel{8}}\cdot\frac{\sqrt{55}}{\cancel{8}\,4} = \frac{3\sqrt{55}}{4}$



$x^2 + 3^2 = 8^2$

$x^2 + 9 = 64$

$x^2 = 55$

$x = \pm\sqrt{55} = \sqrt{55}$ since θ is in quad. I

$\cos\theta = \frac{x}{r} = \frac{\sqrt{55}}{8}$

Bonus. (10pts) Develop the formula for $\cos(4\theta)$ by using sum or double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$\cos(4\theta) = \cos(2\cdot 2\theta) = \cos^2(2\theta) - \sin^2(2\theta)$

$= (\cos(2\theta))^2 - (\sin(2\theta))^2$

$= (\cos^2\theta - \sin^2\theta)^2 - (2\sin\theta\cos\theta)^2$

$= \cos^4\theta - 2\cos^2\theta\sin^2\theta + \sin^4\theta - 4\sin^2\theta\cos^2\theta$

$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$