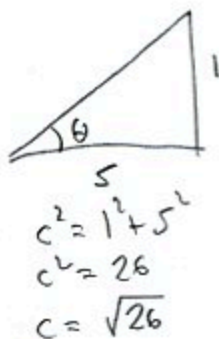


1. (10pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\tan \theta = \frac{1}{5}$. Draw a picture.

$$\tan \theta = \frac{1}{5} = \frac{\text{opp}}{\text{hyp}}$$



$$\cos \theta = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

$$\sec \theta = \frac{\sqrt{26}}{5}$$

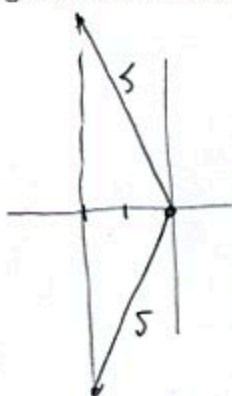
$$\sin \theta = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$

$$\csc \theta = \sqrt{26}$$

$$\tan \theta = \frac{1}{5}$$

$$\cot \theta = 5$$

2. (12pts) If $\cos \theta = -\frac{2}{5}$ and θ is in the third quadrant, find the exact values of all the trigonometric functions of θ . Draw a picture.



$$(-2)^2 + y^2 = 5^2$$

$$4 + y^2 = 25$$

$$y^2 = 21$$

$$y = \pm\sqrt{21}$$

$$y = -\sqrt{21}$$

$$\cos \theta = -\frac{2}{5} = \frac{-2}{5} = \frac{x}{r}$$

due to
3rd quadrant

$$\cos \theta = -\frac{2}{5}$$

$$\sec \theta = -\frac{5}{2}$$

$$\sin \theta = -\frac{\sqrt{21}}{5}$$

$$\csc \theta = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

$$\tan \theta = \frac{-\sqrt{21}}{-2} = \frac{\sqrt{21}}{2}$$

$$\cot \theta = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

3. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

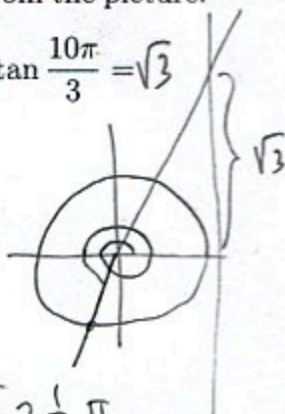
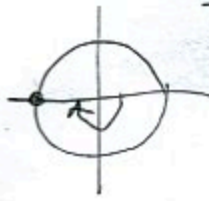
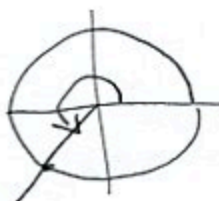
$$\sin 150^\circ = \frac{1}{2}$$

$$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\csc(-180^\circ) = \frac{1}{\sin(-180^\circ)}$$

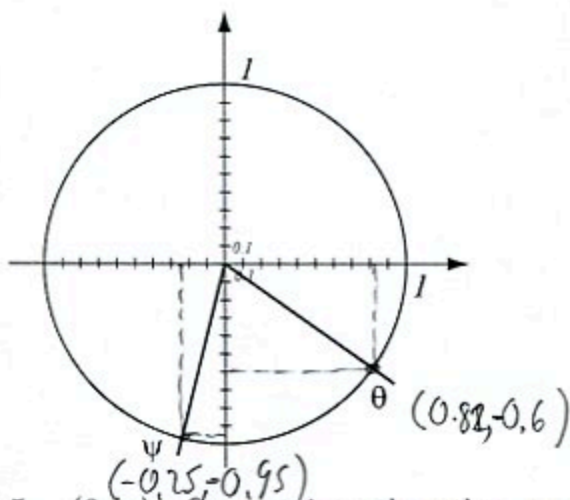
$$\tan \frac{10\pi}{3} = \sqrt{3}$$

$= \frac{1}{0}$ not defined



$$\frac{10\pi}{3} = 3\frac{1}{3}\pi$$

4. (9pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn. Note the angles are **not** the standard angles.



$$\sin \theta = -0.6$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{0.82} = 1.22$$

$$\cos \psi = -0.25$$

$$\tan \psi = \frac{-0.95}{-0.25} = 3.8$$

5. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$$20^\circ = 20^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{9} = 0.349066 \text{ radians}$$

$$\frac{13\pi}{12} \text{ radians} = \frac{13\pi}{12} \cdot \frac{180^\circ}{\pi} = \frac{13 \cdot 180}{12} = 195^\circ$$

6. (6pts) Use your calculator to evaluate (round to 6 decimals):

$$\tan 49^\circ = -1.150368$$

$$\sec \frac{2\pi}{7} = \frac{1}{\cos \frac{2\pi}{7}} = 1.603875$$

7. (3pts) Use your calculator to find the acute angle θ (in degrees, round to 6 decimals) if

$$\sin \theta = \frac{4}{17}$$

$$\theta = \sin^{-1} \frac{4}{17} = 13.608961^\circ$$

8. (10pts) Draw two periods of the graph of $y = 2 \sin(4x + \pi)$.

What is the amplitude? The period?

For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

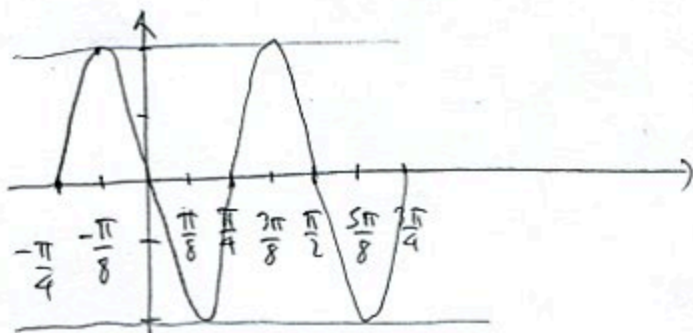
$$\text{amplitude} = 2$$

$$\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

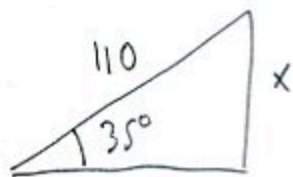
$$-\frac{c}{b} = -\frac{\pi}{4}$$

$$-\frac{c}{b} + \frac{2\pi}{b} = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

$$\text{Tricks at } \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$$



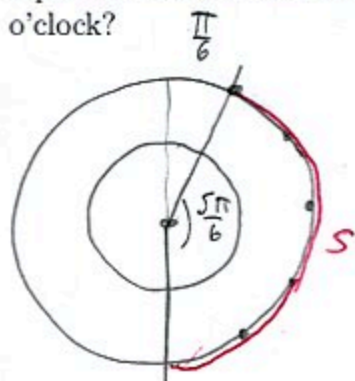
9. (10pts) A kite attached to a 110 ft string is flying so that the angle of elevation from the ground anchor to the kite is 35° . How high above the ground is the kite?



$$\frac{x}{110} = \sin \theta$$

$$x = 110 \cdot \sin \theta = 63.093408 \text{ ft}$$

10. (10pts) Apple's new headquarters building is in the shape of a ring with outer diameter 460 meters. If we refer to points on the circle via correspondence to a clock, how far would a person have to walk along the outside wall to get from a point at 1 o'clock to a point at 6 o'clock?



$$s = r \cdot \theta$$

$$s = 230 \cdot \frac{5\pi}{6} = 602.138592 \text{ meters}$$

$$r = \frac{460}{2} = 230$$

11. (12pts) The Earth rotates around the sun on an approximately circular path of radius 91.4 million miles. It takes the Earth 365.25 days for one complete revolution (hence the leap years!).

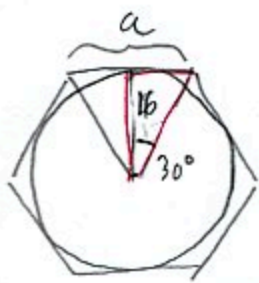
a) What is Earth's angular velocity due to this rotation in radians per hour?

b) What is Earth's linear velocity due to this rotation in miles per hour?

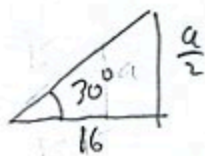
$$\begin{aligned} a) \quad \omega &= \frac{2\pi}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} \\ &\approx 7.16767 \times 10^{-4} \\ &\approx 0.000716767 \text{ radians/hr} \end{aligned}$$

$$\begin{aligned} b) \quad v &= r \cdot \omega \\ &= 91.4 \times 10^6 \cdot 7.16767 \times 10^{-4} \\ &\approx 655.125038 \times 10^2 \\ &\approx 65,512.5038 \text{ mph.} \end{aligned}$$

Bonus. (10pts) A circle of radius 16 meters is inscribed in a regular hexagon. Find the exact value of the perimeter of the hexagon (not a calculator approximation).



$$P = 6 \cdot a$$



$$\frac{360^\circ}{6} = 60^\circ$$

$$\frac{60^\circ}{2} = 30^\circ$$

$$\frac{a/2}{16} = \tan 30^\circ \quad | \cdot 16$$

$$\frac{a}{2} = 16 \tan 30^\circ \quad | \cdot 2$$

$$a = 32 \tan 30^\circ$$

$$\begin{aligned} P &= 6 \cdot 32 \tan 30^\circ \\ &= 192 \cdot \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} &= \frac{192}{\sqrt{3}} = \frac{64 \cdot 3}{\sqrt{3}} = 64\sqrt{3} \text{ meters} \\ &\quad (110.851252) \end{aligned}$$