1. (14pts) Let A, B and C be subsets of some universal set U.
a) Use Venn diagrams to draw the following subsets (shade).
b) Among the four sets, two are equal. Use set algebra to show they are equal.

$(A \cap B) - C$ $\qquad\qquad$ $C - (A \cap B)$ $\qquad\qquad$ $(C - A) \cup (C - B)$ $\qquad\qquad$ $(A \cup B) \cap C$

2. (6pts) Draw arrow diagrams between two sets that illustrate

a) a bijection $\quad$ b) a surjection that is not an injection $\quad$ c) an f where range $f \neq$ codom f

3. (12pts) Let U be the set of integers. Consider the sets $A = \{k \in \mathbf{Z} \mid k \equiv 3 \pmod 5\}$, $B = \{k \in \mathbf{Z} \mid k \text{ is even}\}$, $C = \{k \in \mathbf{Z} \mid -20 \le k \le 20\}$ and write the following subsets using the roster method (pattern needs to be obvious).

$A \cap B$ $\qquad\qquad$ $A - B$ $\qquad\qquad$ B^c $\qquad\qquad$ $A \cap (B \cup C)$ $\qquad\qquad$ $C - (A \cup B)$ $\qquad\qquad$ $B - A$

4. (14pts) Let $A = \{n \in \mathbf{N} \mid n$ is a sum of three consecutive natural numbers$\}$ and $B = \{n \in \mathbf{N} \mid n$ is divisible by 3)$\}$.
a) Is $A \subseteq B$? Prove or disprove.
b) Is $B \subseteq A$? Prove or disprove.

5. (12pts) Let $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ be given by $f(m, n) = 2m - 3n$.
a) Evaluate $f(0, 7)$ and $f(1, -3)$.
b) Determine the set of preimages of 4. List at least three elements of this set and illustrate it in the plane.

6. (16pts) Let $\mathbf{Z}_5 = \{0, 1, 2, 3, 4\}$, and let $f : \mathbf{Z}_5 \to \mathbf{Z}_5$, $g : \mathbf{Z} \to \mathbf{Z}_5$, $f(x) = g(x) = 3x + 7 \pmod 5$. Note that f and g have the same formula, but different domains.
a) Write the table of function values for f.
b) Calculate $g(8)$, $g(-4)$ and $g(100)$.
c) What is the set of preimages of 3 under f?
d) What is the set of preimages of 3 under g? Justify.
e) Is f injective? Justify.
f) Is g injective? Justify.

7. (12pts) Let $f(x) = \dfrac{2x}{x+5}$ and assume the codomain is $\mathbf{R}$.
a) What subset of real numbers is the natural domain for this function?
b) What is the range of this function? Justify your answer.

8. (14pts) Let A, B be subsets of a universal set U. Prove that $A \subseteq B$ if and only if $A \cap B^c = \emptyset$.

Bonus. (10pts) Let S be the set of all functions $f : (0,1) \to \mathbf{R}$ that are differentiable on $(0,1)$, and let T be the set of all functions $g : (0,1) \to \mathbf{R}$. Let $D : S \to T$ be the function of differentiation, that is, $D(f) = f'$.
a) If $f(x) = x^2 - 3x$, find $D(f)$.
b) What is the set of preimages of g, $g(x) = x^3 - 7x$?
c) What is the set of preimages of h, $h(x) = 1$ for $x \in \left(0, \frac{1}{2}\right]$, and $h(x) = -1$ for $x \in \left(\frac{1}{2}, 0\right)$?