Mathematical Reasoning — Exam 2
MAT 312, Fall 2015 — D. Ivanšić

Name: _____

Show all your work!

1. (10pts) Prove the transitive property for congruences (n is a natural number): for all integers a, b and c, if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$.

2. (16pts) Prove using induction: for every natural number n, $1+3+5+7+\cdots+(2n-1) = n^2$.

3. (12pts) Let p be a rational number. Prove: for every real number x, if x is irrational, then $\dfrac{1}{p+x}$ is irrational.

4. (22pts) Consider the statement: for every integer n, n is divisible by 5 if and only if $n^2 + n$ is divisible by 5.
a) Write the statement as a conjunction of two conditional statements.
b) Determine whether each of the conditional statements is true, and write a proof, if so.
c) Is the original statement true?

5. (20pts) Prove the following:
a) For every integer a, if a^2 is divisible by 6, then a is divisible by 6.
b) $\sqrt{6}$ is an irrational number. (Use statement a)).

6. (6pts) Use the triangle inequality to prove that for all real numbers c, d,

$|c + 1 - (d + 3)| < |c - d| + 2$.

7. (14pts) Prove that for all real numbers a, b, $b \neq 0$, $\dfrac{2a}{b} \leq a^2 + \dfrac{1}{b^2}$.

Bonus. (10pts) Use the facts that $\sqrt{2}$ is irrational and that $0 < \frac{\sqrt{2}}{2} < 1$ to show that between any two rational numbers a and b there exists an irrational number.