Mathematical Reasoning — Exam 1
MAT 312, Fall 2015 — D. Ivanšić

Name: ____________________

Show all your work!

Consider the following sentences. If a sentence is a statement, determine whether it is true (and justify your answer). If it is an open sentence, find its truth set.

1. (2pts) If the square of an integer is greater than one, then the integer is greater than one.

2. (2pts) The Earth is round or the moon is closer than 1,000 miles away.

3. (3pts) (universal set=$\mathbf{R}$) $3x - 1 = 3(x + 2) - 7$

4. (3pts) (universal set=$\mathbf{Z}$) $|x| \leq 10$ and x is divisible by 4.

5. (4pts) For every $x \in \mathbf{R}$, $x^2 + 6x + 11 > 0$.

Negate the following statements.

6. (3pts) President Xi supports building a Disney resort in China or puts an order for 300 Boeing airplanes.

7. (3pts) If the Pope says to look after the poor, then I will volunteer for a charity.

8. (6pts) Use a truth table to prove that $(P \implies Q) \vee (Q \implies P)$ is a tautology. (Use however many columns you need.)

P	Q							
T	T							
T	F							
F	T							
F	F							

9. (12pts) Use previously proven logical equivalences to prove the equivalence
$P \vee (Q \implies R) \equiv (Q \implies P) \vee R$. Do not use a truth table.

10. (4pts) Write the converse and contrapositive of the statement: if an integer is divisible by 6, then it is divisible by 3.

Converse:

Contrapositive:

11. (10pts) Suppose the following statements are true:
If Benny gets a ball, then he doesn't get a bicycle.
If Benny gets a bicycle, then he gets a helmet.
Benny didn't get a helmet and he didn't get a ball.

Determine truth value of the following statements and justify.

Benny got a ball.

Benny got a bicycle.

12. (4pts) Use set builder notation to write the set $\{-1, 8, -27, 64, -125, \dots\}$.

13. (7pts) A function $f : (a, b) \to \mathbf{R}$ is said to be differentiable on (a, b) if $f'(c)$ exists for every c in (a, b).
a) Write the definition using symbols.
b) Negate the definition using symbols.
c) Finish the sentence: "A function f is not differentiable on (a, b) if ..."

14. (10pts) For every natural number m, there exists a natural number n such that mn is a square of a natural number.
a) Write this statement using symbols.
b) Write the negation of the statement using symbols.
c) Write the negation of the statement in English.

15. (12pts) Let $\mathbf{R}$ be the universal set. The following is an open sentence in x:

$$(\forall y \in \mathbf{R})(x + y^2 \geq 7)$$

a) If $x = 3$, is the statement true?
b) If $x = 8$, is the statement true?
c) Find the truth set (the x's) of the above statement.

16. (15pts) We will call an integer n type-0, type-1, type-2,..., or type-9 if it can be written in the form $n = 10k$, $n = 10k + 1$, $n = 10k + 2, \ldots$, or $n = 10k + 9$, respectively, for some integer k. Use this idea to show the following: if an integer m ends with 4, and an integer n ends with 7, then the integer $m^2 + mn + n$ ends with 1. Start with a know-show table if you find it helpful.

Bonus. (10pts) Show by example that quantifiers are not "commutative." That is, give an example of an open sentence $P(x, y)$ about real numbers x and y, so that the statements below have opposite truth values (justify why they do).

$(\forall x \in \mathbf{R})(\exists y \in \mathbf{R})P(x, y)$ $\qquad\qquad$ $(\exists y \in \mathbf{R})(\forall x \in \mathbf{R})P(x, y)$