Mathematical Reasoning — Handout
MAT 312, Fall 2017 — D. Ivanšić

Review Sheet

Sections 1.1, 1.2, 2.1–2.4

1.1 Sentences and statements (truth value of)
Conditional Statements
Closure properties of number systems
Understanding when the statement "If P, then Q" is true

1.2 Definition of even and odd integer
Know-show table
Proof-writing guidelines (read again)
Constructing simple proofs involving integers

2.1 Truth tables for $\neg P$, $P \wedge Q$, $P \vee Q$, $P \Longrightarrow Q$
Showing equivalence using truth tables
Other forms of the conditional statement (language)
Biconditional statement $P \Longleftrightarrow Q$

2.2 Converse and contrapositive of a statement
Theorem 2.8: established logical equivalences, know all except last two
Negating statements in words
Determining logical equivalences using established ones

2.3 Sets, set notation, roster method, set builder notation
Open sentences (predicates), their truth sets, and finding them

2.4 Quantifiers "for every" and "there exists"
Turning statements with $\forall$, $\exists$ into English and vice-versa
Negations of quantified sentences with one or more quantifiers
Converting statements with more than one quantifier
from symbols to English and vice-versa.
Determining truth of statements with one or more quantifiers.

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Sections 3.1–3.5, 4.1, 4.2

3.1 Divisibility and congruence (definitions and manipulating statements)

3.2 Proving a statement $P \implies Q$ by proving the contrapositive $\neg Q \implies \neg P$
Proving biconditional statements (prove both directions)
Proofs by construction (explicitly producing the object whose existence is claimed)
Proofs without construction (showing an object exists, without knowing what it is)

3.3 Proofs by contradiction
Know to use contrapositive instead of contradiction when convenient
Avoid contradiction when a proof can be done directly (often with inequalities)

3.4 Proofs that are broken up into cases by, for example:
- odd and even integers
- remainders when divided by a certain number
- positive and negative real numbers, etc.

Basic properties of absolute value and triangle inequality (3.23 and 3.25)

3.5 Division algorithm and congruences
Know how to manipulate congruences:
- congruences may be added
- congruences may be multiplied
- congruences may be raised to a power

Applications to problems concerning divisibility (often times by contrapositive)

4.1 Definition of an inductive set
Know principle of mathematical induction
Prove various statements using induction, like in homework

4.2 Know other forms of mathematical induction:
- basis step may be a number other than 1
- assumption of induction step is that statement is valid for numbers $1, 2, \ldots, k$

Prove various statements using induction, like in homework