1. (8pts) Let $A = (-1, 3)$ and $B = (7, -3)$.
a) Find the midpoint M of A and B.
b) Verify that the distances from M to A and M to B are equal.

2. (10pts) Write the equation of the circle with center $(-3, -4)$ that contains the point $(2, 1)$. Sketch the circle.
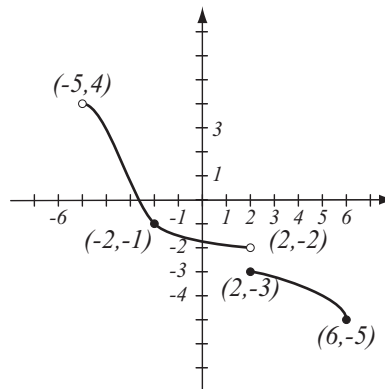
3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(-2)$ and $f(2)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions
   of the equation $f(x) = -4$?

(-5,4)

3

1

-6   -1   1 2 3 4 5 6

(-2,-1) -2   (2,-2)
         -3
         -4   (2,-3)

(6,-5)

4. (12pts) The function
$f(x) = -x^4 + 5x^3 + 4x^2 - 7x - 8$ is
given.

a) Use your calculator to accurately
its graph. Draw the graph here, and
indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts
(accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt[3]{3x - 17}}{2x - 5}$$

$$g(x) = \frac{\sqrt{x}}{x^2 - 5x - 24}$$

6. (10pts) Let $g(x) = \dfrac{3x + 5}{x^2 - 9}$. Find the following (simplify where appropriate).

$g(3) =$

$g(1) =$

$g(-z) =$

$g(w + 3) =$

1. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(2, -5)$ and is perpendicular to the line $2x - 3y = 6$. Draw both lines.

2. (5pts) Find the linear function f its x-intercept is 3 and $f(5) = -2$.

3. (9pts) Draw the triangle with vertices $A = (-3, 0)$, $B = (1, 5)$, and $C = (6, 1)$. Use slopes to determine if the triangle is a right triangle.

4. (4pts) The consumption of gasoline in the U.S. has varied over the years. In 2012, 3.178 billion barrels were consumed; in 2016, 3.413 billion barrels of gasoline were consumed. What is the average rate of change of gasoline consumed from 2012 to 2016? What are the units for the average rate of change?

5. (12pts) A business that manufactures decorative mirrors keeps track of its expenses. One month, it produced 213 mirrors and had expenses of $6463.36. Another month, it produced 327 mirrors and had expenses of $9395.44.

a) Assuming that the business expenses $E(x)$ is a linear function of the number of mirrors x produced, write a formula for $E(x)$.

b) What are the expenses if no mirrors are produced? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A farmer is trying to establish the relationship between the amount of rainfall during growing season and the yield of corn on his farm. The table shows the data: W is the amount of rainfall in millimeters and Y is the corn yield in tons. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between W and Y. Draw the line on the graph.

c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.

d) Find the coefficient of correlation r. How strong is the linear relationship between W and Y?

e) What yield can the farmer expect if the the amount of rainfall in a year is 750 millimeters?

W	Y
540	10
558	10.7
620	10.9
700	11.2
790	11
825	12
900	13.5

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $1 \leq 3 - 4x < 8$

2. (7pts) $3x + 1 < 10$ or $7x - 3 > 25$

3. (6pts) Find the domain of the function in interval notation: $f(x) = \dfrac{\sqrt{8 - 3x}}{4x + 2}$.

4. (14pts) Two computer-maintenance freelancers charge for their services in this way: George charges \$150 for the first three hours and then \$40 per hour for hours after the first three; Yuri charges \$200 for the first five hours and then \$45 per hour for hours after the first five. Assuming at least five hours of work are needed, for which number of hours is it preferable to hire George? Solve as an inequality.

5. (14pts) A 298-mile-long road joins cities Charlestown and Crown City. At the same time, one car leaves Charlestown and drives toward Crown City, and another car, driving 7mph slower than the first car, leaves Crown City and drives toward Charlestown. After 2 hours they meet on the road.
a) What are the speeds of the cars?
b) How far from Charlestown did they meet?

6. (14pts) How many liters of a 3% solution of sulphuric acid must be mixed with 2 liters of a 7% solution of sulphuric acid in order to get a 6% solution of sulphuric acid?

1. (10pts) Use your calculator to accurately sketch the graph of the function
$f(x) = \dfrac{x^2 + 3x + 1}{x^2 + 3}$. Draw the graph here, indicate units on the axes, and solve the problems
below with accuracy 6 decimal points.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \sqrt{x + 3}$, $g(x) = \dfrac{x^2 + 1}{x - 4}$. Find the following (simplify where possible):

$(f - g)(1) =$ $(fg)(6) =$

$\dfrac{g}{f}(x) =$ $(f \circ g)(3) =$

$(g \circ f)(x) =$

The domain of $(fg)(x)$ in interval notation

3. (8pts) Consider the function $h(x) = (x^2 - x)^2$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x + 2, & \text{if } -2 < x \leq 3 \\ 7 - 3x, & \text{if } 3 < x. \end{cases}$$

5. (14pts) A builder is charged with designing a simple school house with area 5000 square feet, four rooms and a corridor at least 6 feet wide. The school board wishes to minimize the cost, which is the same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the school house for which the total length of the walls is minimal? What is the minimal wall length?
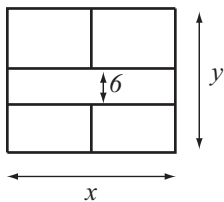
1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - x^2 + 4x \qquad\qquad g(x) = x^5 - 7x^3 \qquad\qquad h(x) = |x| - 3x^2 + 1$$

2. (16pts) Using transformations, draw the graphs of $f(x) = 4 + \sqrt[3]{-x}$ and $g(x) = -3(x-2)^2$. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:

a) shape of $y = \sqrt{x}$, shifted up 2 units

b) shape of $y = x^3$ stretched horizontally by factor $\frac{1}{3}$, then shifted right 4 units

c) shape of $y = \frac{1}{x}$, stretched vertically by factor 4, then reflected about the y-axis, then shifted left 1 unit.

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x-3)$, $2f(-x)$ and $f(2x) + 1$ and label all the relevant points.
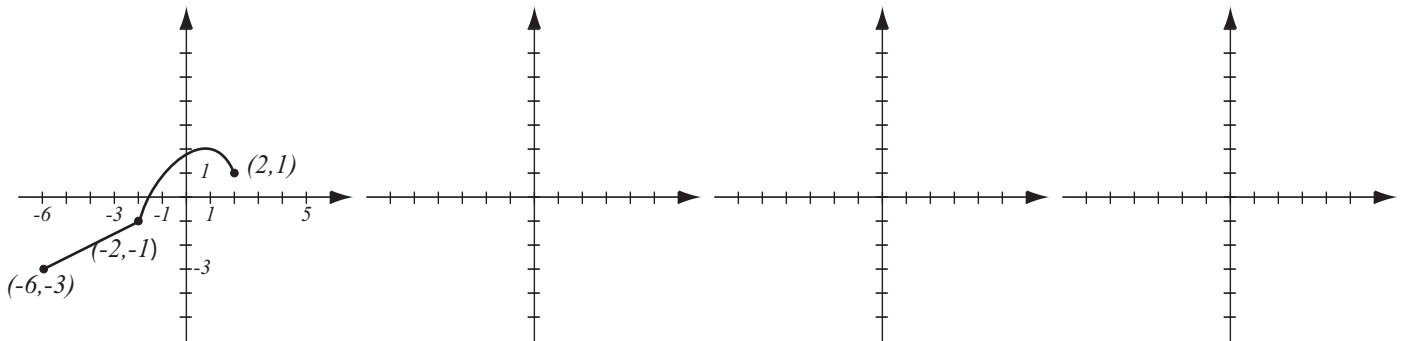
Simplify, so that the answer is in form $a + bi$.

1. (4pts) $4 + (5i - 1) \cdot 2i =$

2. (6pts) $\dfrac{3 + 5i}{2 - 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{103} =$

4. (8pts) The amount of food (in tons) arriving daily to a hurricane-stricken area is given by $A(x) = -x^2 + 20x + 15$, where x is the number of days after September 20th. On what dates were 106 tons arriving daily?

5. (8pts) Solve the equation: $2x^4 + 7x^2 - 15 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 12x + 8 = 0$

7. (12pts) The quadratic function $f(x) = x^2 + 3x - 28$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

8. (12pts) Greg is in charge of irrigating a large field and his weekly pay depends on the weekly rainfall. His base pay of \$11 an hour is supplemented by hardship hourly pay of \$1 per inch of rainfall (for working in a muddy field). However, if it rains, less work on irrigation is needed, so his 40 weekly hours are reduced by 2 hours per inch of rainfall. One week, Greg was paid \$476. How many inches of rain fell during that week?

1. (4pts) Solve the equation.

$|2x - 1| = 4$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x + 8| < 3$ $\qquad\qquad\qquad\qquad$ $|5x - 6| \geq 9$

Solve the equations:

3. (8pts) $\dfrac{2x + 1}{x - 6} - \dfrac{16x + 21}{x^2 - 3x - 18} = \dfrac{x - 4}{x + 3}$ $\qquad$ **4.** (8pts) $3x + \sqrt{26 - 5x} = 2x + 4$

5. (14pts) A ball is thrown upwards from the ground with initial velocity 21 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 21t$.
a) Sketch the graph of the height function.
b) When does the ball reach its greatest height, and what is that height?
c) When is the ball at height 21 meters?

6. (14pts) Truck mechanic Igor wishes to build a repair shop with two side-by-side bays separated by a shorter wall (see picture). Igor has enough money to build 900 feet of walls, and he wants to build a shop with maximal area.
a) Express the total area of the shop as a function of one of the sides of the rectangle. What is the domain of this function?
b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the greatest total area? What is the greatest area possible?
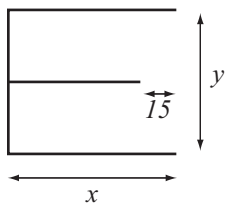
1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph
of f^{-1}, labeling the relevant points.
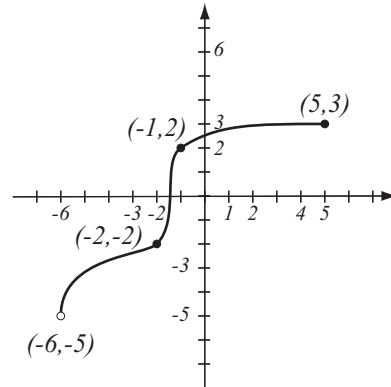
2. (12pts) Let $f(x) = x^2 - 7$, considered for domain $x \leq 0$.
a) Find the formula for f^{-1}.
b) Find the ranges of f and f^{-1}.

3. (8pts) Evaluate without using the calculator. Show how you got the numbers.

$\log_2 64 =$ $\qquad$ $\log_4 \frac{1}{256} =$ $\qquad$ $\log_{27} 9 =$ $\qquad$ $\log_{b^2} \sqrt[3]{b^4} =$

4. (4pts) Use the change-of-base formula
and your calculator to find $\log_4 18$ with ac-
curacy 6 decimal places. Show how you
obtained your number.

5. (7pts) How much should you invest in an account bearing 2.25%, compounded monthly, if you wish to have $3,000 in two years?

6. (6pts) Find the domain of $f(x) = \dfrac{\log_5(3 - 2x)}{6x - 25}$.

7. (8pts) An object weighing 2000 kilograms moving at velocity v meters per second has kinetic energy $E(v) = 1000v^2$ Joules.
a) Determine the kinetic energy of the object if it is traveling at 10 and 20 meters per second.
b) Find a formula for the inverse function and explain what it represents.
c) Determine the velocity of the object if its kinetic energy is 150,000 and 300,000 Joules.

8. (9pts) Using transformations, draw the graph of $f(x) = 7 - e^{0.5x}$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If $\log_a 5 = 1.609438$ and $\log_a 9 = 2.197225$, find (show how you obtained your numbers):

$\log_a \dfrac{5}{9} =$ $\log_a 405 =$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 \left(1024 \sqrt[4]{x^7} y^3 \right) =$

$\log_2 \dfrac{x^4 y^{\frac{2}{3}}}{16\sqrt{x}\sqrt[3]{y^5}} =$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\log(27x^8) - 2\log(6y^{\frac{5}{6}}) + \log y^2 - 3\log x =$

$3\log_9(x+5) - 2\log_9(x^2 + 5x) - 3\log_9 x =$

Solve the equations.

4. (5pts) $16^{2x-1} = \left(\dfrac{1}{2}\right)^{2x+3}$

5. (7pts) $7^{3-x} = 4^{2x-9}$

6. (8pts) $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 4$

7. (12pts) The uranium isotope uranium-232 has a half-life of 68.9 years. Suppose a sample contains 10 grams of uranium-232.
a) Write the function describing the amount $P(t)$ of uranium t years after it starts decaying.
b) Sketch the graph of the function.
c) When will the sample have 2.5 grams of uranium-232?

1. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\sec\theta = \frac{7}{4}$.

2. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\cos\theta = w$, where w is some number.

3. (10pts) Given that $\cos 18° = a$, $\sin 73° = b$, $\tan 22° = c$ and $\csc 61° = d$, use basic and cofunction identities to express the following quantities using a, b, c and d.

$\cos 17° =$ $\csc 73° =$ $\sin 61° =$ $\sec 29° =$

$\sin 72° =$ $\sec 18° =$ $\cot 68° =$ $\cot 22° =$

4. (10pts) Solve the right triangle (that is, find all sides and angles), if $b = 3$, $c = 7$.

5. (8pts) A ladder leans against the wall and forms a 73° angle with the ground. If its bottom is 6 feet from the wall, how long is the ladder?

6. (16pts) Standing at point A, you observe the angle of elevation to the top of a building to be 27°. Then you walk in a straight line through the building, emerging on the other side at point B, and measuring the angle of elevation from this point to the top of the building to be 43°. A and B are known landmarks and the distance between them is 250 meters.
a) How tall is the building?
b) How far is point A from the building?
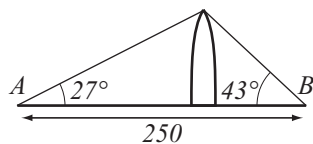
1. (9pts) If $\csc\theta = -\dfrac{6}{5}$ and θ is in the fourth quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

2. (7pts) The terminal side of angle θ is in the third quadrant and lies on the line $4x-7y=0$. Find the exact values of $\cos\theta$ and $\tan\theta$. Draw a picture.

3. (8pts) Sketch angles in standard position with indicated radian measure.
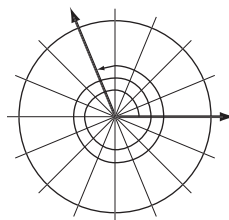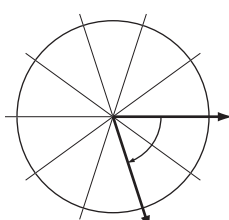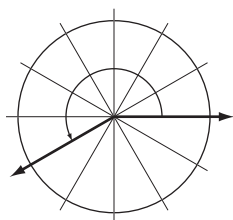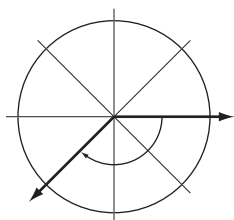
$$\dfrac{4\pi}{3} \qquad\qquad -\dfrac{5\pi}{8} \qquad\qquad -\dfrac{11\pi}{6} \qquad\qquad \dfrac{23\pi}{7}$$

4. (8pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced lines to help you determine what the angles are.)

5. (8pts) A circular irrigation system with radius 1/4 mile rotates 11° in an hour. How far in miles has the part on the rim traveled, if the system has worked 14 hours?

6. (8pts) If we approximate Moon's path by a circle (it is really an ellipse), it rotates around earth along a circle of radius 384,000 kilometers once every 27.321661 days. What is its linear speed in kilometers per second?

7. (12pts) A wheel of radius 4in sits on a conveyor belt, which makes it rotate. The conveyor belt moves at 1/3 mile per hour.
a) What is the angular speed of the wheel, in radians per second?
b) How many revolutions per hour does the wheel make?

1. (10pts) Suppose that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\cos\alpha = \frac{3}{7}$ and $\sin\beta = \frac{1}{5}$. Find the exact value of $\sin(\alpha - \beta)$.

2. (4pts) Use an identity to find the exact value of the expression (do not use the calculator):

$\cos 93° \cos 27° - \sin 93° \sin 27° =$

3. (8pts) Find the exact value of $\cos \dfrac{5\pi}{8}$ (do not use the calculator).

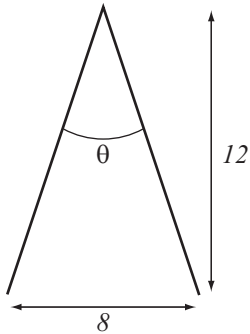**4.** (10pts) Use identities to simplify the following expressions.

$\sin\left(\dfrac{\pi}{3} - \theta\right) \cos\left(\dfrac{\pi}{6} - \theta\right) + \sin\left(\dfrac{\pi}{6} - \theta\right) \cos\left(\dfrac{\pi}{3} - \theta\right) =$

$\dfrac{\tan\left(\dfrac{\pi}{2} - \theta\right)}{\cos\theta} \cdot \dfrac{\tan\theta}{\cos(-\theta)} + \dfrac{\sin(-\theta)}{\cos\left(\dfrac{\pi}{2} - \theta\right)} =$

5. (8pts) Show the identity.

$$\frac{\sin(u - v)}{\cos u \cos v} = \tan u - \tan v$$

6. (10pts) A 12-foot tall roof is 8 feet wide. Find the exact value for $\cos\theta$ (do not use the calculator), where θ is the angle the roof subtends.

7. (10pts) Develop the formula for $\cos(3\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$$\cos(3\theta) = \cos(2\theta + \theta) =$$