Algebra and Trigonometry — Joysheet 12	Name: Saul Ocean
MAT 150, Fall 2017 — D. Ivanšić	*Covers: 7.1-7.3* *Show all your work!*

1. (10pts) Suppose that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\cos\alpha = \frac{3}{7}$ and $\sin\beta = \frac{1}{5}$. Find the exact value of $\sin(\alpha - \beta)$.

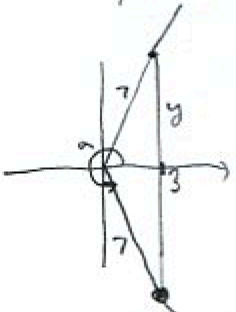
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = -\frac{2\sqrt{10}}{7}\cdot\left(-\frac{2\sqrt{6}}{5}\right) - \frac{3}{7}\cdot\frac{1}{5} = \frac{4\sqrt{60}-3}{35} = \frac{8\sqrt{15}+3}{35}$

$\cos\alpha = \frac{3}{7} = \frac{x}{r}$, $\sin\alpha = \frac{y}{r} = -\frac{2\sqrt{10}}{7}$

$\sin\beta = \frac{1}{5} = \frac{y}{r}$

$x^2 + 1^2 = 5^2$ $\cos\beta = -\frac{2\sqrt{6}}{5}$

$x^2 = 24$

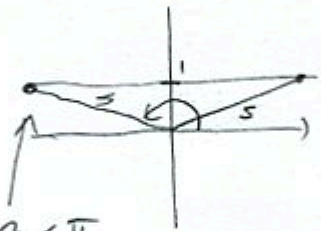


$3^2 + y^2 = 7^2$

$y^2 = 40$

$y = \pm\sqrt{40}$

$y = -2\sqrt{10}$



$\frac{\pi}{2} < \beta < \pi$

$x = \pm\sqrt{24}$

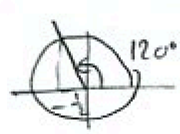
$x = -2\sqrt{6}$

since $\frac{\pi}{2} < \beta < \pi$

IV since $\frac{3\pi}{2} < \alpha < 2\pi$

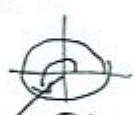
2. (4pts) Use an identity to find the exact value of the expression (do not use the calculator):

$\cos 93° \cos 27° - \sin 93° \sin 27° =$ $\cos(93° + 27°) = \cos 120° = -\frac{1}{2}$



3. (8pts) Find the exact value of $\cos\frac{5\pi}{8}$ (do not use the calculator).

$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

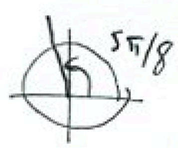


$\cos\frac{5\pi}{8} = \pm\sqrt{\frac{2-\sqrt{2}}{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2}$

$\cos^2\left(\frac{1}{2}\cdot\frac{5\pi}{4}\right) = \frac{1+\cos\frac{5\pi}{4}}{2}$

since $\frac{5\pi}{8}$ is in Q2

$\cos^2\frac{5\pi}{8} = \frac{1-\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2} = \frac{2-\sqrt{2}}{4}$



4. (10pts) Use identities to simplify the following expressions.

$\sin\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{6}-\theta\right) + \sin\left(\frac{\pi}{6}-\theta\right)\cos\left(\frac{\pi}{3}-\theta\right) = \sin\left(\frac{\pi}{3}-\theta+\frac{\pi}{6}-\theta\right) = \sin\left(\frac{\pi}{2}-2\theta\right)$

$= \cos(2\theta) = \cos^2\theta - \sin^2\theta$

product is 1

$\frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cos\theta}\cdot\frac{\tan\theta}{\cos(-\theta)} + \frac{\sin(-\theta)}{\cos\left(\frac{\pi}{2}-\theta\right)} = \frac{\cot\theta}{\cos\theta}\cdot\frac{\tan\theta}{\cos\theta} + \frac{-\sin\theta}{\sin\theta} = \frac{1}{\cos^2\theta} - 1 = \sec^2\theta - 1$

$= \tan^2\theta$

odd/even
cofunction identities

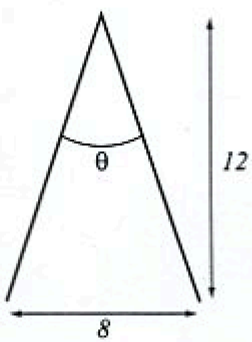
5. (8pts) Show the identity.

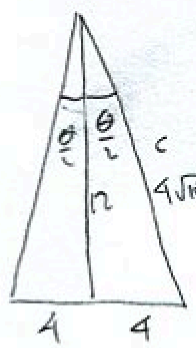
$$\frac{\sin(u-v)}{\cos u \cos v} = \tan u - \tan v$$

$$\frac{\sin(u-v)}{\cos u \cos v} = \frac{\sin u \cos v - \cos u \sin v}{\cos u \cos v} = \frac{\sin u \cancel{\cos v}}{\cos u \cancel{\cos v}} - \frac{\cancel{\cos u} \sin v}{\cancel{\cos u} \cos v}$$

$$= \frac{\sin u}{\cos u} - \frac{\sin v}{\cos v} = \tan u - \tan v$$

6. (10pts) A 12-foot tall roof is 8 feet wide. Find the exact value for $\cos\theta$ (do not use the calculator), where θ is the angle the roof subtends.





$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos\frac{\theta}{2} = \frac{12}{4\sqrt{10}}$$

$$\text{so } \cos\theta = 2\cos^2\frac{\theta}{2} - 1$$

$$= 2\cdot\left(\frac{\cancel{12}\,3}{\cancel{4}\sqrt{10}}\right)^2 - 1$$

$$= \cancel{2}\cdot\frac{9}{\cancel{10}\,5} - 1 = \frac{9}{5} - 1 = \frac{4}{5}$$

$$c^2 = 4^2 + 12^2 \qquad c = 4\sqrt{10}$$

$$c^2 = 160$$

$$c = \pm\sqrt{160}$$

7. (10pts) Develop the formula for $\cos(3\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$$\cos(3\theta) = \cos(2\theta + \theta) = \cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta$$

$$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\cdot\sin\theta$$

$$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$$

$$= \cos^3\theta - 3\sin^2\theta\cos\theta$$