Algebra and Trigonometry — Joysheet 9
MAT 150, Fall 2017 — D. Ivanšić

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Covers: 5.4–5.6 *Show all your work!*

1. (5pts) If $\log_a 5 = 1.609438$ and $\log_a 9 = 2.197225$, find (show how you obtained your numbers):

$\log_a \frac{5}{9} = \log_a 5 - \log_a 9$
$= 1.609438 - 2.197225$
$= -0.587787$

$\log_a 405 = \log_a 5 \cdot 81 = \log_a 5 \cdot 9^2$
$= \log_a 5 + \log_a 9^2$
$= \log_a 5 + 2\log_a 9$
$= 1.609438 + 2 \cdot 2.197225 = 6.003888$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4\left(1024\sqrt[4]{x^7}y^3\right) = \log_4 1024 + \log_4 x^{\frac{7}{4}} + \log_4 y^3$
$x^{\frac{7}{4}}$
$= 5 + \frac{7}{4}\log_4 x + 3\log_4 y$

$\log_2 \frac{x^4 y^{\frac{2}{3}}}{16\sqrt{x}\sqrt[3]{y^5}} = \log_2 x^4 + \log_2 y^{\frac{2}{3}} - \log_2 16 - \log_2 x^{\frac{1}{2}} - \log_2 y^{\frac{5}{3}}$
$x^{\frac{1}{2}}$ $y^{5/3}$
$= 4\log_2 x + \frac{2}{3}\log_2 y - 4 - \frac{1}{2}\log_2 x - \frac{5}{3}\log_2 y$
$= \frac{7}{2}\log_2 x - \log_2 y - 4$
$4 - \frac{1}{2} = \frac{7}{2}$ $\frac{2}{3} - \frac{5}{3} = -1$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\log(27x^8) - 2\log(6y^{\frac{5}{6}}) + \log y^2 - 3\log x = \log(27x^8)^{\frac{1}{3}} - \log(6y^{\frac{5}{6}})^2 + \log y^2 - \log x^3$
$= \log(3x^{\frac{8}{3}}) - \log(36y^{\frac{5}{3}}) + \log y^2 - \log x^3$
$= \log \frac{3x^{\frac{8}{3}}y^2}{36y^{\frac{5}{3}}x^3} = \log \frac{x^{-\frac{1}{3}}y^{\frac{1}{3}}}{12} = \log \frac{y^{\frac{1}{3}}}{12x^{\frac{1}{3}}}$
$\frac{8}{3} - 3$ $2 - \frac{5}{3}$

$3\log_9(x+5) - 2\log_9(x^2+5x) - 3\log_9 x =$
$= \log_9(x+5)^3 - \log_9(x^2+5x)^2 - \log_9 x^3$
$= \log_9 \frac{(x+5)^3}{(x(x+5))^2 x^3} = \log_9 \frac{(x+5)^3}{(x+5)^2 x^5} = \log_9 \frac{x+5}{x^5}$

Solve the equations.

4. (5pts) $16^{2x-1} = \left(\frac{1}{2}\right)^{2x+3}$

$(2^4)^{2x-1} = (2^{-1})^{2x+3}$

$2^{8x-4} = 2^{-2x-3}$

$8x-4 = -2x-3$

$10x = 1,\ x = \frac{1}{10}$

5. (7pts) $7^{3-x} = 4^{2x-9}$ $\mid \ln$

$\ln 7^{3-x} = \ln 4^{2x-9}$

$(3-x)\ln 7 = (2x-9)\ln 4$

$3\ln 7 - x\ln 7 = 2\ln 4 \cdot x - 9\ln 4$

$2\ln 4 \cdot x + \ln 7 \cdot x = 3\ln 7 + 9\ln 4$

$x(2\ln 4 + \ln 7) = 3\ln 7 + 9\ln 4$

$x = \frac{3\ln 7 + 9\ln 4}{2\ln 4 + \ln 7} = 3.8814$

6. (8pts) $\frac{e^x + e^{-x}}{e^x - e^{-x}} = 4$ $\mid \cdot\, e^x - e^{-x}$

$e^x + e^{-x} = 4(e^x - e^{-x})$ $\mid -e^x - e^{-x}$

$3e^x - 5e^{-x} = 0$ $\mid \cdot e^x$

$3e^{2x} - 5 = 0$

$3e^{2x} = 5$

$e^{2x} = \frac{5}{3}$ $\mid \ln$

$2x = \ln\frac{5}{3}$

$x = \frac{\ln\frac{5}{3}}{2} = 0.255413$

7. (12pts) The uranium isotope uranium-232 has a half-life of 68.9 years. Suppose a sample contains 10 grams of uranium-232.
a) Write the function describing the amount $P(t)$ of uranium t years after it starts decaying.
b) Sketch the graph of the function.
c) When will the sample have 2.5 grams of uranium-232?

a) $P(t) = 10e^{-kt}$

$5 = 10e^{-k \cdot 68.9}$ $\mid \div 10$

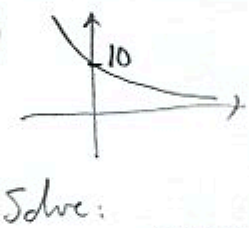
$\frac{1}{2} = e^{-68.9k}$ $\mid \ln$

$\ln\frac{1}{2} = -68.9k$

$k = \frac{\ln\frac{1}{2}}{-68.9} = \frac{\ln 2}{68.9} = 0.0100602$

$P(t) = 10e^{-0.0100602t}$

b)



c) Solve:

$2.5 = 10e^{-0.0100602t}$ $\mid \div 10$

$\frac{1}{4} = e^{-0.010\ldots t}$ $\mid \ln$

$\ln\frac{1}{4} = -0.010\ldots t$

$t = \frac{\ln\frac{1}{4}}{-0.0100602} = 137.8$

Or:

2.5 is a quarter of 10. After 68.9 years, $\frac{1}{2}$ of U-232 is left. After another 68.9 years half of that half is left, so one quarter. Thus, $t = 2 \cdot 68.9 = 137.8$