Algebra and Trigonometry — Joysheet 5
MAT 150, Fall 2017 — D. Ivanšić

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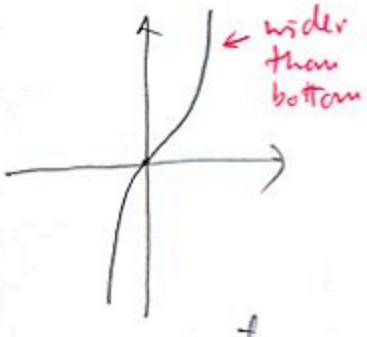
Covers: 2.4, 2.5 *Show all your work!*

1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^3 - x^2 + 4x$

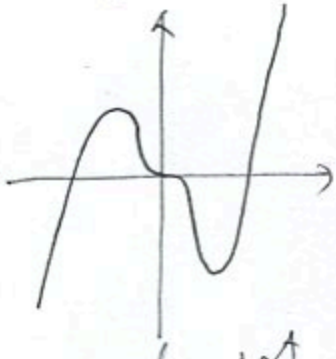
$f(-x) = (-x)^3 - (-x)^2 + 4(-x)$
$= -x^3 - x^2 - 4x$
$\neq f(x), -f(x)$
so neither



no symmetry

$g(x) = x^5 - 7x^3$

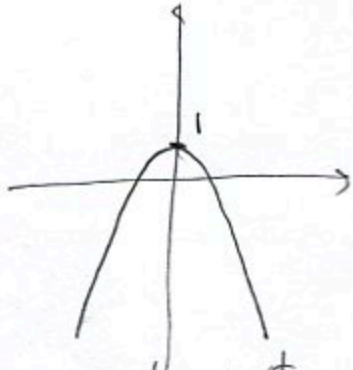
$g(-x) = (-x)^5 - 7(-x)^3$
$= -x^5 - 7(-x^3)$
$= -x^5 + 7x^3$
$= -g(x)$ odd



symmetric wrt origin

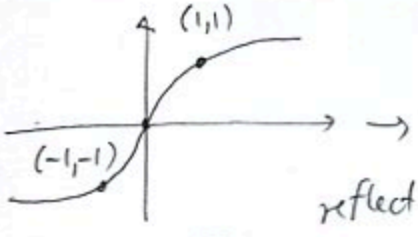
$h(x) = |x| - 3x^2 + 1$

$h(-x) = |-x| - 3(-x)^2 + 1$
$= |x| - 3x^2 + 1$
$= h(x)$ even

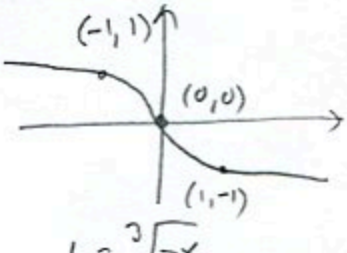


symmetric wrt y-axis

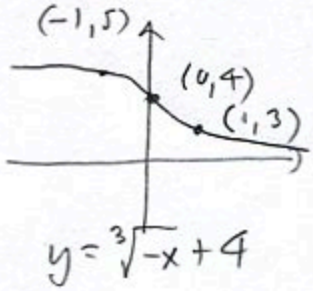
2. (16pts) Using transformations, draw the graphs of $f(x) = 4 + \sqrt[3]{-x}$ and $g(x) = -3(x-2)^2$. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.



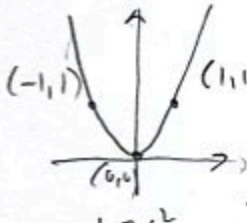
$y = \sqrt[3]{x}$ → reflect in y-axis



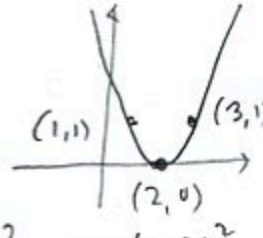
$y = \sqrt[3]{-x}$ → shift up 4



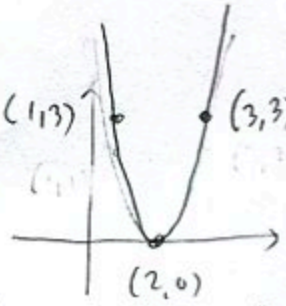
$y = \sqrt[3]{-x} + 4$



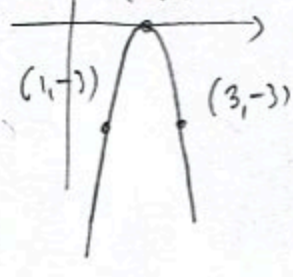
$y = x^2$ → shift right 2



$y = (x-2)^2$ → stretch vertically, factor = 3



$y = 3(x-2)^2$ → reflect in y-axis



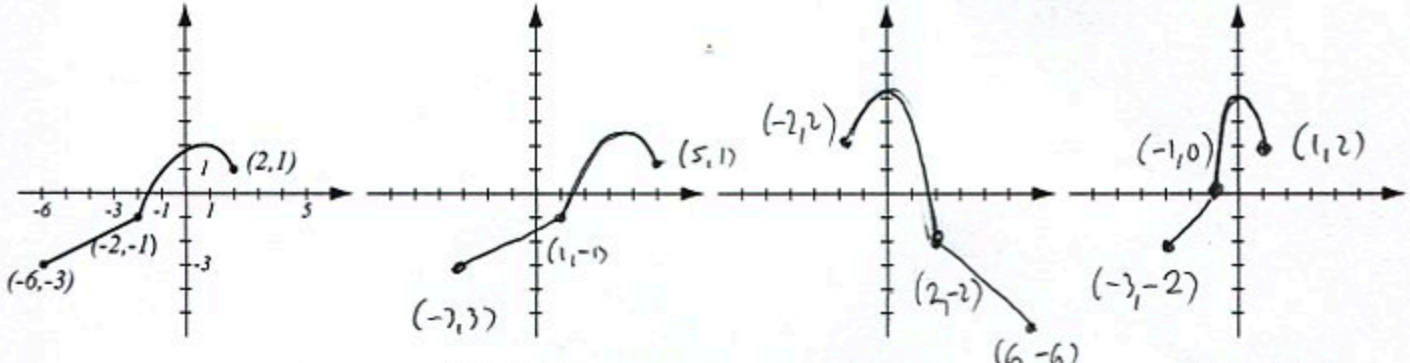
3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, shifted up 2 units
b) shape of $y = x^3$ stretched horizontally by factor $\frac{1}{3}$, then shifted right 4 units
c) shape of $y = \frac{1}{x}$, stretched vertically by factor 4, then reflected about the y-axis, then shifted left 1 unit.

a) $y = \sqrt{x} + 2$

b) $y = x^3 \rightarrow y = (3x)^3 \rightarrow y = (3(x-4))^3 = 27(x-4)^3$

c) $y = 4 \cdot \frac{1}{x} = \frac{4}{x} \rightarrow y = \frac{4}{-x} = -\frac{4}{x} \rightarrow y = -\frac{4}{x+1}$

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x-3)$, $2f(-x)$ and $f(2x)+1$ and label all the relevant points.



$f(x-3)$
shift right 3 units

$2f(-x)$
reflect in y-axis, stretch vertically, factor = 2

$f(2x)+1$
stretch horizontally, factor = $\frac{1}{2}$, shift up 1