Algebra and Trigonometry — Exam 1
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

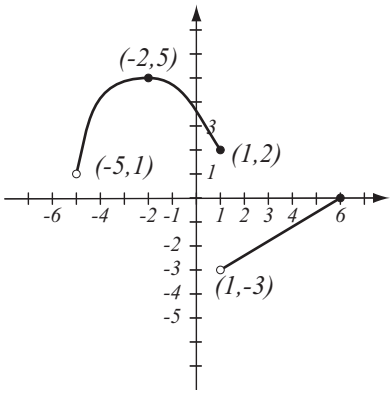
1. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-2) =$ $f(1) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 2$?



2. (5pts) Write the equation of a linear function f for which $f(-1) = 4$ and $f(3) = -2$.

3. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(2, 3)$ and is perpendicular to the line $x - 2y = 6$. Draw both lines.

4. (9pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{2}{x^2 + 3x - 10} \qquad g(x) = \frac{\sqrt{10 - 2x}}{x - 7}$$

5. (19pts) The function $f(x) = x^4 - 2x^2 + 9x - 7$ is given. Solve with accuracy 6 decimal points.
a) Use your calculator to accurately draw its graph on paper. Indicate units on the axes.
b) Find all the $x-$ and $y-$intercepts.
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.
e) State the domain and range.

6. (9pts) Let $g(x) = \dfrac{2x-3}{x^2+2x+1}$. Find the following (simplify where appropriate).

$g(1) =$ $g(-1) =$

$g(-x) =$ $g(u-3) =$

7. (4pts) A household spent 22,110 kWh of electricity in 2012. After installing efficient appliances and lighting, they spent 19,673 kWh in 2016. What is the average rate of change of electricity consumption from 2012 to 2016? What are the units for the average rate of change?

8. (10pts) Let $A = (0, -3)$, $B = (1, 2)$ and $C = (3, -1)$.
a) Draw the three points and show algebraically that the distance from A to C is the same as the distance from B to C.
b) This means that A and B lie on a circle whose center is C. Write the equation of this circle and draw it in the picture.

9. (12pts) Linda has these options for a data plan for her cell phone:
A) \$18 flat fee for the first two GB, and then \$7 per GB for usage beyond the first two GB.
B) \$8 per GB.
Assuming Linda always uses at least 2 GB of data, for which amount of data is plan B better?

10. (14pts) A truck drives a heavy load from a warehouse to a store at 56mph. After unloading, the lighter truck is now able to make the return trip driving at 64mph. Ignoring time spent at the store, the total time spent driving to the store and back was 3 hours.
a) How long did the truck drive to the store? From the store?
b) How far is the store?

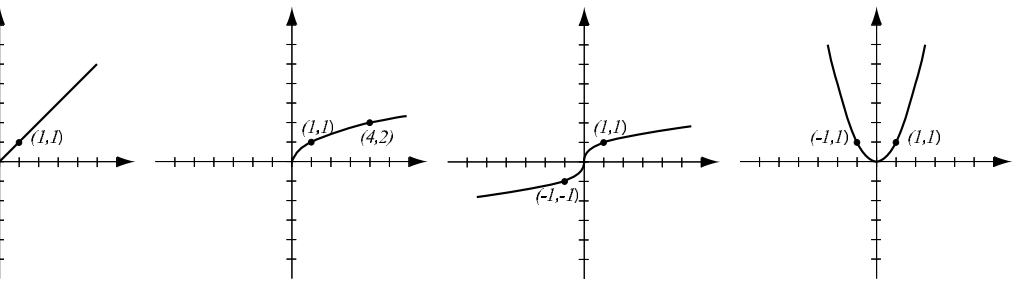
Bonus. (10pts) A 4-liter jug contains 1 liter of a 30% solution of muriatic acid. You have pure water and an 8% solution of muriatic acid. How much of each should you add to the jug to end up with a full jug of a 12% solution of muriatic acid? *(Hint: think of this problem in the usual way, as mixing three containers to get a fourth with a 12% solution.)*

Algebra and Trigonometry — Exam 2
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



Simplify, so that the answer is in form $a + bi$.

2. (4pts) $\dfrac{3+7i}{2i} =$

3. (4pts) Simplify and justify your answer.

$i^{86} =$

4. (17pts) Let $f(x) = \dfrac{x-1}{x^2-4}$, $g(x) = \sqrt{x} - 3$.

Find the following (simplify where possible):

$(fg)(4) =$ $(g \circ f)(1) =$

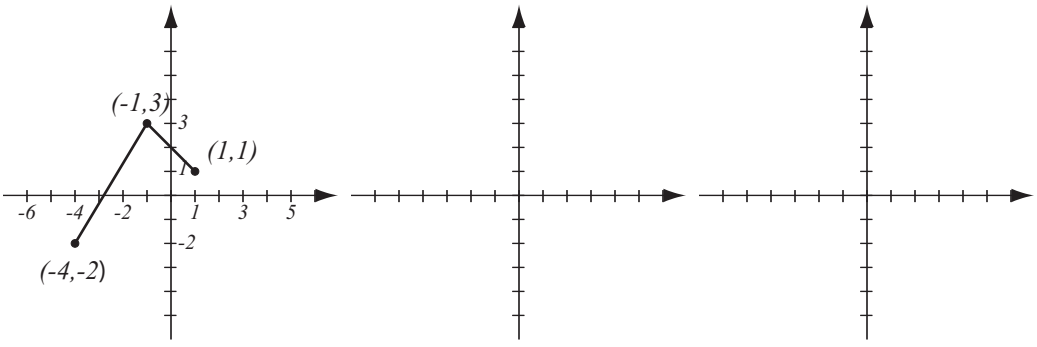
$\dfrac{g}{f}(x) =$ $(f \circ g)(x) =$

The domain of $f - g$ in interval notation

5. (12pts) The quadratic function $f(x) = x^2 + 6x + 10$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

6. (6pts) Consider the function $h(x) = \sqrt{x^2 - 3x + 5}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-3)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



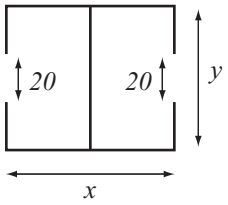
8. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 20 = 0$

9. (7pts) For the function $f(x) = x^4 - 6x^2 + 2$:
a) determine algebraically whether it is odd, even, or neither
b) use the calculator to draw its graph here and verify your conclusion by stating symmetry.

10. (12pts) Starting with a square, we decrease the width by 1 inch and increase the length by 5 inches to arrive at a rectangle. Starting with the same square, we decrease the width by 2 inches and increase the length by 22 inches to arrive at another rectangle whose area is twice the area of the earlier rectangle. How long is the side of the square?

11. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 800 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Among all rectangles with diagonal of length 5 centimeters, find the dimensions of the one with the greatest perimeter. What is the greatest possible perimeter?

Algebra and Trigonometry — Exam 3
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

1. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x-4|>3$ $|3x+13|\leq 8$

Solve the equations:

2. (8pts) $\dfrac{7x-51}{x^2+9x+14}+\dfrac{2x-6}{x+7}=\dfrac{x}{x+2}$

3. (8pts) $x+\sqrt{51+10x}=-6$

4. (8pts) Evaluate without using the calculator. Show how you got the numbers.

$\log_8 64=$ $\log_3 \frac{1}{27}=$ $\log_8 16=$ $\log_{\sqrt[3]{b}} b^2=$

5. (4pts) Use the change-of-base formula and your calculator to find $\log_{11} 27$ with accuracy 6 decimal places. Show how you obtained your number.

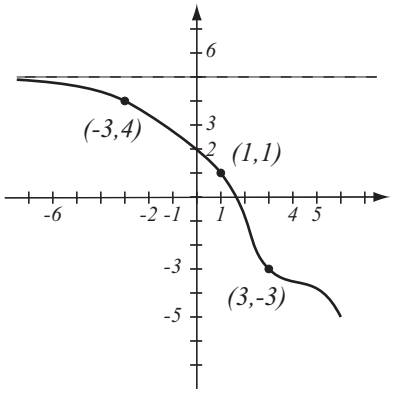
6. (3pts) For the polynomial $P(x) = 4x^5 - 8x^3 + 4x^2 - 7x + 1$ state:

degree: leading coefficient: leading term:

7. (14pts) The polynomial $f(x) = (x-5)^2(x+3)(x+1)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

8. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of x-intercepts and turning points.
b) Draw a graph of P that has exactly 2 x-intercepts.
c) Draw a graph of P that has no turning points.

9. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



10. (9pts) Let $f(x) = \dfrac{5}{x^3 - 2}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

11. (6pts) Using transformations, draw the graph of $f(x) = 4\ln(x + 3)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

12. (8pts) Find the domain of the function $f(x) = \dfrac{\log_5(3x+8)}{|x+4|-2}$ and write it in interval notation.

13. (8pts) How much should you invest in an account bearing 2.3%, compounded daily, if you wish to have \$2,500 in five years?

Bonus. (10pts) Let $f(x) = x^2 + 2x - 4$ with domain $x \leq -1$.
a) Sketch the graph of the function. Is it a one-to-one function?
b) Find $f^{-1}(x)$. *(Hint: quadratic formula.)*

Algebra and Trigonometry — Exam 4
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

1. (5pts) If $\log_a 2 = u$ and $\log_a 3 = v$, express in terms of u and v:

$\log_a 6 =$

$\log_a \dfrac{3}{8} =$

2. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \dfrac{\sqrt[5]{u^4}}{49w^3} =$

3. (6pts) Write as a single logarithm. Simplify if possible.

$2\ln(x^3y^{-2}) - 3\ln(x^{-2}y^5) =$

Solve the equations.

4. (6pts) $9^{2x-7} = \left(\dfrac{1}{3}\right)^{2x+6}$

5. (8pts) $5^{2x+1} = 4^{-3x+4}$

6. (12pts) The population of Expandaton was 127,000 in 2012 and 154,000 in 2017. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2012. Graph it on paper.
b) Find the predicted population in the year 2021.

7. (10pts) If $\sin\theta = \frac{1}{5}$ and θ is in the second quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

8. (6pts) Sketch angles in standard position with indicated radian measure.

$$\frac{7\pi}{6} \qquad -\frac{3\pi}{8} \qquad \frac{13\pi}{5}$$

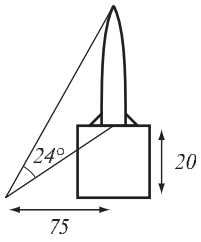
9. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$9^\circ =$

$\frac{7\pi}{15}$ radians $=$

10. (9pts) An airplane is flying at altitude 3 miles when it spots a city in the distance. If the angle of depression to the city is 42°, what is the line-of-sight (through the air) distance from the airplane to the city?

11. (14pts) From a point on the ground 75 meters away from the launch pad, you observe a rocket and note it subtends an angle of 24°. If the launch pad is 20 meters tall, how tall is the rocket?



12. (12pts) Our textbook has dimensions approximately 9×11 inches. Skillful student Fred is rotating it on his finger at 50 revolutions per minute. (His finger touches the center of the book.)
a) What is the angular velocity of this rotation in radians per second?
b) What is the linear velocity of the corner of the book, in inches per second?

Bonus. (10pts) Let θ be the latitude of a point on Earth, R the radius of the Earth. The point circles the axis of Earth's rotation on a circle of a radius r (the farther north the point, the smaller the r). Write the formula for r using R and θ.

Algebra and Trigonometry — Exam 5
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

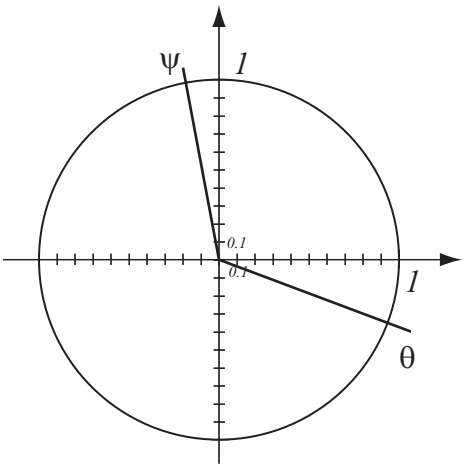
Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\cos 150^\circ =$ $\qquad$ $\sin \dfrac{3\pi}{4} =$ $\qquad$ $\csc(-120^\circ) =$ $\qquad$ $\tan \dfrac{5\pi}{2} =$

2. (8pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn. Note the angles are **not** the standard angles.



$\sin\theta =$ $\qquad$ $\tan\theta =$

$\cos\psi =$ $\qquad$ $\csc\psi =$

3. (8pts) Use identities to simplify the following expression.

$$\sin\left(\frac{\pi}{2} - \theta\right)\cos\theta - \cos\left(\frac{\pi}{2} - \theta\right)\sin(-\theta) =$$

4. (14pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\cos \dfrac{5\pi}{12} =$

$\sin 202.5^\circ =$

Show the identities:

5. (8pts) $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta} = 2\sec\theta\tan\theta$

6. (8pts) $\dfrac{\sin\theta\cos\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)} = \dfrac{1}{2}\tan(2\theta)$

7. (12pts) Suppose that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\pi < \beta < \frac{3\pi}{2}$ are angles so that $\cos\alpha = \frac{2}{5}$ and $\sin\beta = -\frac{3}{4}$. Using identities (sum, difference, half- or double-angle) and without using the calculator, find the exact value of $\tan(\alpha + \beta)$.

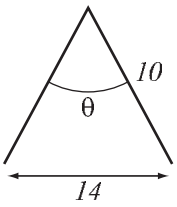
8. (10pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos\dfrac{\sqrt{2}}{2} =$ $\qquad$ $\arctan\left(-\dfrac{1}{\sqrt{3}}\right) =$ $\qquad$ $\arccos\left(-\dfrac{\sqrt{3}}{2}\right) =$ $\qquad$ $\arcsin 2 =$

9. (8pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\cos(\arccos 0.3) =$ $\qquad$ $\arcsin\left(\sin\dfrac{7\pi}{5}\right) =$

10. (12pts) A 10-ft folding ladder is placed on a floor so that its ends are 14 feet apart. Find the exact value for $\sin\theta$ (do not use the calculator), where θ is the angle the ladder subtends.



Bonus. (10pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\cos\left(2\arcsin\left(-\frac{3}{7}\right)\right) =$$

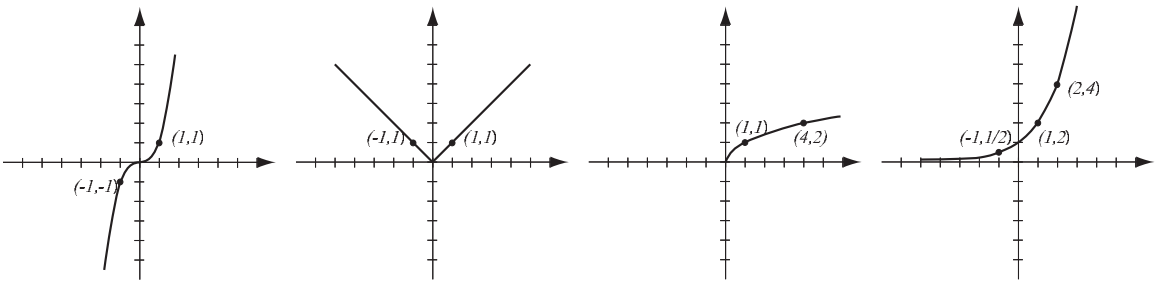
Algebra and Trigonometry — Final Ex.
MAT 150, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



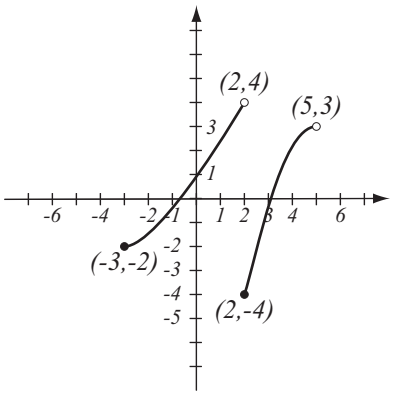
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-3) =$ $\qquad f(2) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 1$?



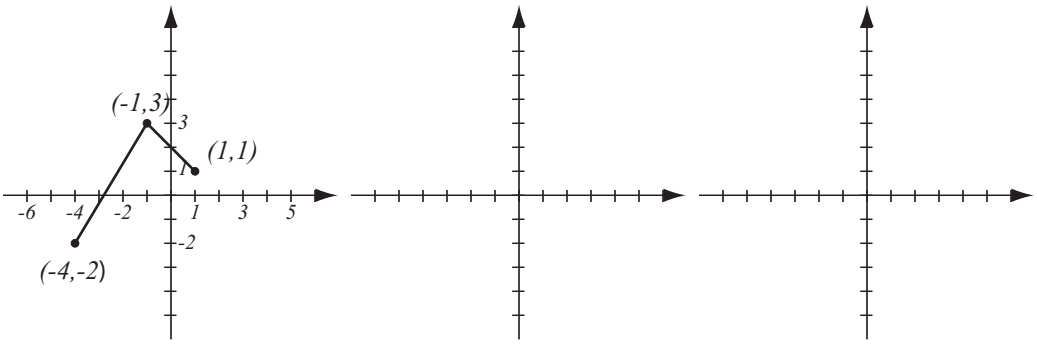
3. (9pts) Write the equation of the line whose x-intercept is -3 and passes through $(1, 2)$. Is this line perpendicular to the line $x + 2y = 7$? Draw both lines.

4. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 3| < 4$

5. (6pts) Find the domain of the function $f(x) = \dfrac{\ln(5 - 2x)}{x^2 - 3x - 18}$ and write it in interval notation.

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x - 3)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



7. (19pts) The polynomial $P(x) = x^4 - 13x^2 + 36$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

8. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \frac{x^2}{81\sqrt[4]{y^7}} =$$

9. (5pts) Write as a single logarithm. Simplify if possible.

$$\log(x^3y^{-5}) - 4\log(xy^{-2}) =$$

Solve the equations.

10. (8pts) $x + \sqrt{4x + 17} = 1$

11. (8pts) $3^{2x+1} = 4^x$

12. (12pts) Without using the calculator, find the exact values of the following trigonometric functions or their inverses. Draw the unit circle and the appropriate picture to infer the values from the picture.

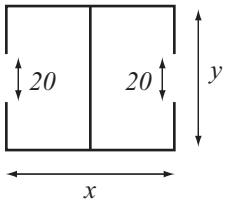
$\sin 210^\circ =$ $\tan \dfrac{3\pi}{4} =$ $\arcsin \dfrac{1}{2} =$ $\arccos\left(-\dfrac{\sqrt{2}}{2}\right) =$

13. (10pts) Suppose that $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\tan\alpha = \frac{2}{5}$ and $\cos\beta = -\frac{4}{9}$. Using identities (sum, difference, half- or double-angle) and without using the calculator, find the exact value of $\cos(\alpha + \beta)$.

14. (8pts) An airplane is flying at altitude 2 miles when it spots a city in the distance. If the angle of depression to the city is 24°, what is the line-of-sight (through the air) distance from the airplane to the city?

15. (14pts) A truck drives a heavy load from a warehouse to a store at 40mph. After unloading, the lighter truck is now able to make the return trip driving at 60mph. Ignoring time spent at the store, the total time spent driving to the store and back was 2 hours.
a) How long did the truck drive to the store? From the store?
b) How far is the store?

16. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1400 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Let $f(x) = x^2 + 2x - 4$ with domain $x \leq -1$.
a) Sketch the graph of the function. Is it a one-to-one function?
b) Find $f^{-1}(x)$. *(Hint: quadratic formula.)*