| **Trigonometry — Exam 3** | **Name:** |
| **MAT 145, Spring 2017— D. Ivanšić** | *Show all your work!* |

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ $\sin(2u) = 2 \sin u \cos u$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ $\cos(2u) = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$

$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$

$\cos^2 \frac{u}{2} = \frac{1 + \cos u}{2}$ $\sin^2 \frac{u}{2} = \frac{1 - \cos u}{2}$ $\tan^2 \frac{u}{2} = \frac{1 - \cos u}{1 + \cos u}$

1. (6pts) Solve the triangle: $a = 8$, $b = 3$, $c = 4$.

2. (14pts) Solve the triangle: $b = 10$, $c = 7$, $B = 44°$

3. (13pts) Solve the triangle: $b = 3$, $c = 2$, $A = 79°$.

4. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$$(r, \theta) = \left(2, \frac{5\pi}{6} \right)$$

$$(r, \theta) = \left(-5, -\frac{3\pi}{4} \right)$$

5. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$$(x, y) = (3, -3)$$

$$(x, y) = (-2\sqrt{3}, 2)$$

6. (9pts) Convert to a polar equation. Answer should be solved for r.

$$x^2 + 2xy + y^2 = 5$$

7. (8pts) The vertices of a triangle are given in **polar coordinates**: $A = (0,0)$, $B = \left(4, \frac{\pi}{3}\right)$, $C = \left(5, \frac{\pi}{2}\right)$.
a) Draw the triangle.
b) Find the exact area of the triangle (do not use the calculator).

**8.** (8pts) Use your calculator to draw an accurate graph of the polar curve $r = 1 + 5\cos(4\theta)$.

9. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is $63°$ and the angle between lines of sight BA and BC from point B is $46°$.
a) How far apart are A and C?
b) How far apart are B and C?
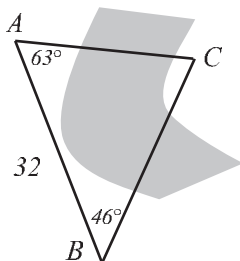
10. (13pts) Two planes leave an airport: one flies $N12°W$ at 450 mph, and the other flies $S37°E$ at 540 mph. What is the distance c between the planes after two hours?
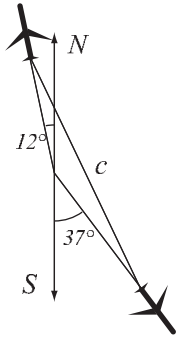
Bonus. (10pts) In a circle of radius a, the large triangle, whose bottom side is a diameter, is split into two triangles as shown.
a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.