Trigonometry — Exam 2
MAT 145, Spring 2017— D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (16pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\sin 75^\circ =$

$\tan 157.5^\circ =$

2. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin \frac{1}{2} =$ $\qquad \arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\qquad \arcsin(4) =$ $\qquad \arctan \frac{1}{\sqrt{3}} =$

3. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\sin(\arcsin(-0.4)) =$ $\qquad \arccos\left(\cos \frac{9\pi}{7}\right) =$

4. (7pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\cos\left(\arctan\left(-\frac{7}{4}\right)\right) =$$

5. (8pts) Use identities to simplify the following expression.

$$\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\theta}+\cos\left(\frac{\pi}{2}-\theta\right)\sin(-\theta) =$$

Show the identities:

6. (8pts) $\tan\theta(\tan\theta+\cot\theta)=\sec^2\theta$

7. (8pts) $(\sin\theta+\cos\theta)^2=1+\sin(2\theta)$

8. (5pts) Solve the equation in radians (give a general formula for all solutions).

$2\cos\theta + \sqrt{3} = 0$

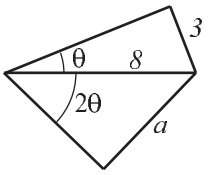
9. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.8$

10. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta + \cos\theta - 1 = 0$

11. (12pts) The two triangles in the picture are right triangles. One of them has an angle of measure θ, the other, 2θ. Find the exact value for the length of side a (do not use the calculator).



Bonus. (10pts) Develop the formula for $\cos(4\theta)$ by using sum or double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.