Algebra and Trigonometry — Final Ex.
MAT 150, Fall 2017 — D. Ivanšić

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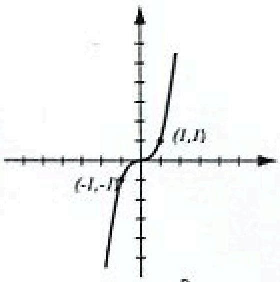
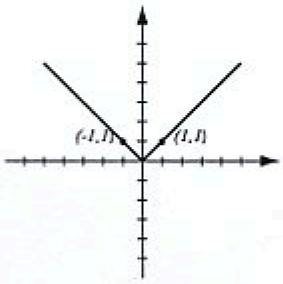
Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

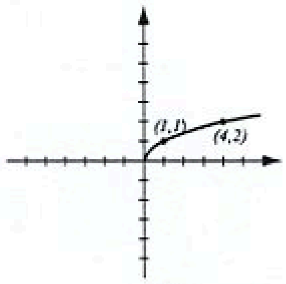
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



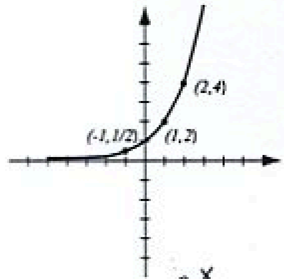
$y = x^3$



$y = |x|$

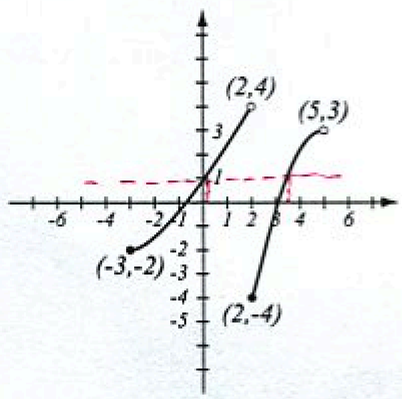


$y = \sqrt{x}$



$y = 2^x$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-3) = -2$ $\quad f(2) = -4$

b) What is the domain of f? $[-3, 5)$

c) What is the range of f? $[-4, 4)$

d) What are the solutions of the equation $f(x) = 1$? $x = 0, 3.4$

3. (9pts) Write the equation of the line whose x-intercept is -3 and passes through $(1, 2)$. Is this line perpendicular to the line $x + 2y = 7$? Draw both lines.

Passes through $(-3,0)$ and $(1,2)$

$m = \frac{2-0}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - (-3))$

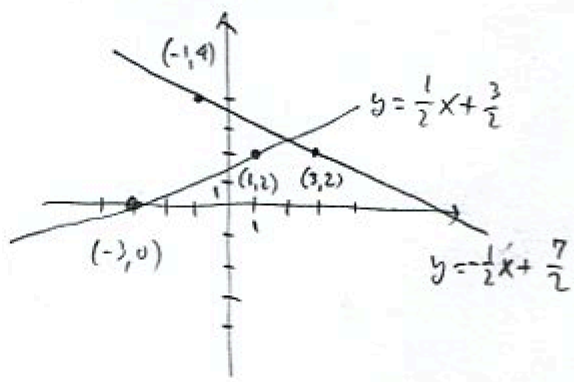
$y = \frac{1}{2}x + \frac{3}{2}$

$x + 2y = 7$

$2y = -x + 7 \mid \div 2$

$y = -\frac{x}{2} + \frac{7}{2}$

Slopes $\frac{1}{2}$ and $-\frac{1}{2}$ are not opposite reciprocal, so lines are not perpendicular

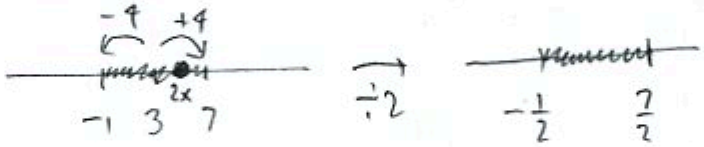


4. (6pts) Solve the inequality. Write the solution in interval form.

$|2x-3| < 4$

distance from $2x$ to $3 < 4$

$\left[-\frac{1}{2}, \frac{7}{2}\right]$



5. (6pts) Find the domain of the function $f(x) = \dfrac{\ln(5-2x)}{x^2-3x-18}$ and write it in interval notation.

Must have: $5-2x > 0$ and Can't have: $x^2-3x-18=0$

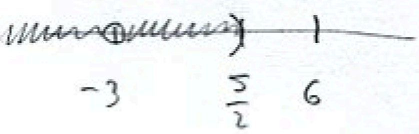
$5 > 2x$

$x < \frac{5}{2}$

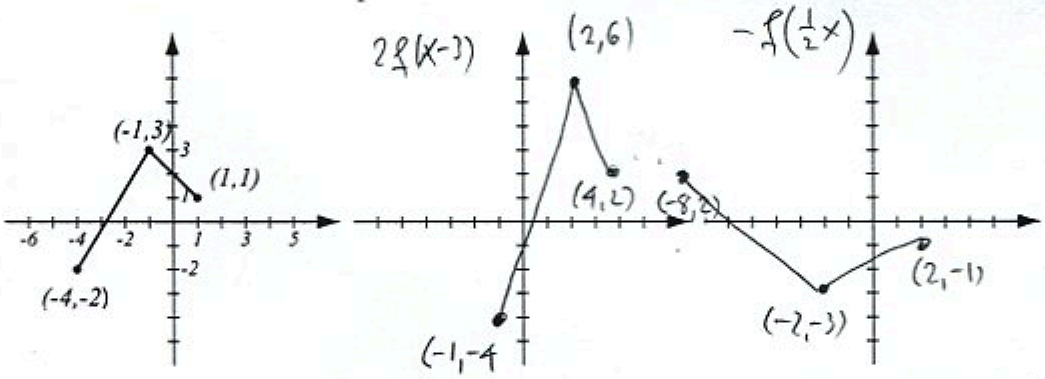
$(x-6)(x+3)=0$

$x = 6, -3$

Domain: $(-\infty,-3) \cup (-3, \frac{5}{2})$



6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-3)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



Vertical stretch, factor 2

shift right 3

Horizontal stretch, factor = 2

Reflect in y-axis

7. (19pts) The polynomial $P(x) = x^4 - 13x^2 + 36$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) Like x^4 ⋃

b) $x^4 - 13x^2 + 36 = [u = x^2]$

$= u^2 - 13u + 36$

$= (u-4)(u-9)$

$= (x^2-4)(x^2-9)$

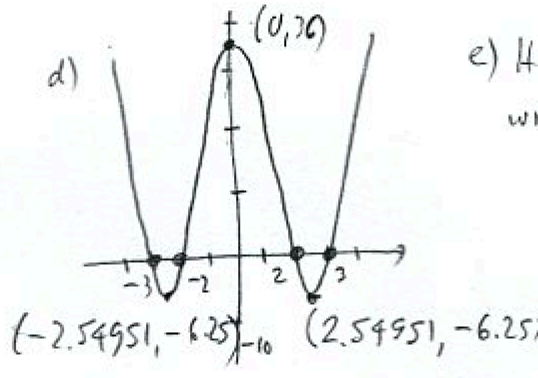
$= (x-2)(x+2)(x-3)(x+3)$

zero	2	-2	3	-3
mult.	1	1	1	1

y-int: 36.

c) $P(-x) = (-x)^4 - 13(-x)^2 + 36$
$= x^4 - 13x^2 + 36 = P(x)$, so even

d)



e) It is symmetric wrt y-axis

f) Local max is $f(0) = 36$
Local minima are: $f(-2.54951, -6.25)$
$f(-2.54951, -6.25)$

8. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \frac{x^2}{81\sqrt[4]{y^7}} = \log_3 x^2 - \log_3 81 - \log_3 y^{\frac{7}{4}}$

$y^{\frac{7}{4}}$

$= 2\log_3 x - 4 - \frac{7}{4}\log_3 y$

9. (5pts) Write as a single logarithm. Simplify if possible.

$\log(x^3y^{-5}) - 4\log(xy^{-2}) = \log(x^3y^{-5}) - \log(xy^{-2})^4$

$= \log \frac{x^3y^{-5}}{(xy^{-2})^4} = \log \frac{x^3y^{-5}}{x^4y^{-8}}$

$= \log(x^{-1}y^3) = \log \frac{y^3}{x}$

Solve the equations.

10. (8pts) $x+\sqrt{4x+17}=1 \quad |-x$

$\sqrt{4x+17}=1-x \quad |^2$

$4x+17=1-2x+x^2 \quad |-4x-17$

$x^2-6x-16=0$

$(x+2)(x-8)=0$

$x=-2,\ 8 \qquad \boxed{x=-2}$

Check: $-2+\sqrt{4\cdot(-2)+17}=1$

$-2+\sqrt{9}=1$ yes

$8+\sqrt{4\cdot 8+17}=1$

$8+\sqrt{49}=1$ no

11. (8pts) $3^{2x+1}=4^x \quad |\ln$

$\ln 3^{2x+1}=\ln 4^x$

$(2x+1)\ln 3=x\ln 4$

$2x\ln 3+\ln 3=x\ln 4$

$2x\ln 3-x\ln 4=-\ln 3$

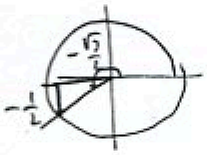
$x(2\ln 3-\ln 4)=-\ln 3$

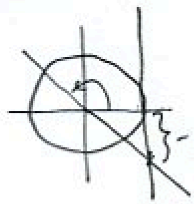
$x=\dfrac{-\ln 3}{2\ln 3-\ln 4}=\dfrac{\ln 3}{\ln 4-2\ln 3}=-1.354756$

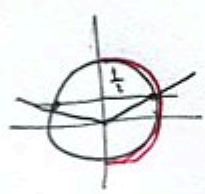
12. (12pts) Without using the calculator, find the exact values of the following trigonometric functions or their inverses. Draw the unit circle and the appropriate picture to infer the values from the picture.

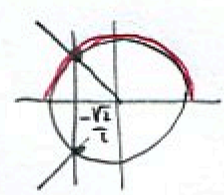
$\sin 210^\circ = -\frac{1}{2}$ $\qquad$ $\tan\frac{3\pi}{4} = -1$ $\qquad$ $\arcsin\frac{1}{2}=\frac{\pi}{6}$ $\qquad$ $\arccos\left(-\frac{\sqrt{2}}{2}\right)=\frac{3\pi}{4}$

$\sin\theta=\frac{1}{2}$

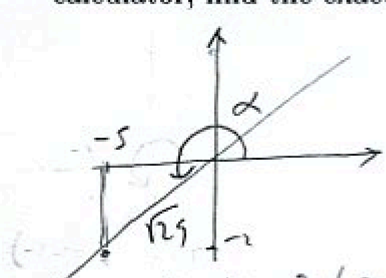








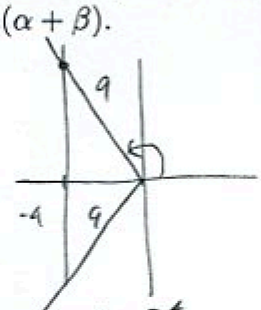
13. (10pts) Suppose that $\frac{\pi}{2}<\alpha<\frac{3\pi}{2}$ and $\frac{\pi}{2}<\beta<\pi$ are angles so that $\tan\alpha=\frac{2}{5}$ and $\cos\beta=-\frac{4}{9}$. Using identities (sum, difference, half- or double-angle) and without using the calculator, find the exact value of $\cos(\alpha+\beta)$.



$\tan\alpha=\frac{2}{5}=\frac{y}{x}=\frac{-2}{-5}$ (so it is in Q3)

$r=\sqrt{(-2)^2+(-5)^2}=\sqrt{29}$

$\sin\alpha=-\frac{2}{\sqrt{29}},\ \cos\alpha=\frac{-5}{\sqrt{29}}$



$\cos\beta=\frac{-4}{9}$

$(-4)^2+y^2=9^2$

$y^2=65,\ y=\pm\sqrt{65}=\sqrt{65}$ due to Q2

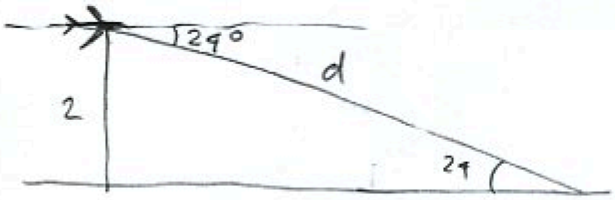
$\sin\beta=\frac{\sqrt{65}}{9}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=-\frac{5}{\sqrt{29}}\cdot\left(-\frac{4}{9}\right)-\left(-\frac{2}{\sqrt{29}}\right)\cdot\frac{\sqrt{65}}{9}$

$=\frac{20}{9\sqrt{29}}+\frac{2\sqrt{65}}{9\sqrt{29}}=\boxed{\dfrac{20+2\sqrt{65}}{9\sqrt{29}}}$

14. (8pts) An airplane is flying at altitude 2 miles when it spots a city in the distance. If the angle of depression to the city is 24°, what is the line-of-sight (through the air) distance from the airplane to the city?



$$\frac{2}{d} = \sin 24^\circ$$

$$\frac{2}{\sin 24^\circ} = d$$

$d \approx 4.917187$ miles

15. (14pts) A truck drives a heavy load from a warehouse to a store at 40mph. After unloading, the lighter truck is now able to make the return trip driving at 60mph. Ignoring time spent at the store, the total time spent driving to the store and back was 2 hours.
a) How long did the truck drive to the store? From the store?
b) How far is the store?

$d, 40, t$ $\longrightarrow$

$\longleftarrow$ $d, 60, 2-t$

$d = 40t$

$d = 60(2-t)$

$40t = 60(2-t)$

$40t = 120 - 60t$

$100t = 120$

$t = 1.2$ hrs

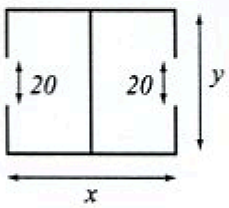
a) Drove 1.2 hours to store
Drove 0.8 hours from store

b) $d = 40 \cdot 1.2 = 48$ miles

16. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1400 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



a) $A = xy = \left(720 - \frac{3}{2}y\right)y = -\frac{3}{2}y^2 + 720y$

$1400 = 2x + y + 2(y-20)$

$1400 = 2x + 3y - 40$

$2x = 1440 - 3y$

$x = 720 - \frac{3}{2}y$

Domain:
Must have:

$y \geq 20$

$x \geq 0$

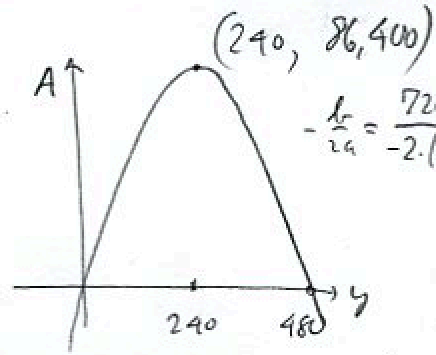
$720 - \frac{3}{2}y \geq 0$

$\frac{3}{2}y \leq 720 \quad | \cdot \frac{2}{3}$

$y \leq 480$

Domain:

$[20, 480]$



$-\frac{b}{2a} = \frac{720}{-2\cdot\left(-\frac{3}{2}\right)} = 240$

$720 - \frac{3}{2}\cdot 240$

dimensions: 240×360 ft

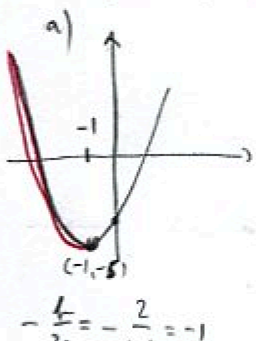
Max area is 86400 sq. ft

Bonus. (10pts) Let $f(x) = x^2 + 2x - 4$ with domain $x \leq -1$.

a) Sketch the graph of the function. Is it a one-to-one function?

b) Find $f^{-1}(x)$. *(Hint: quadratic formula.)*

a)



$-\frac{b}{2a} = -\frac{2}{2\cdot 1} = -1$

$(-1)^2 + 2\cdot(-1) - 4 = -6$

Red part is graph of f
- passes horiz. line test,
so it is a one-to-one
function.

b) $y = x^2 + 2x - 4$

$x^2 + 2x \underbrace{-4 - y}_{\text{"c"}} = 0$ solve for x

$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-4-y)}}{2\cdot 1}$

$= \frac{-2 \pm \sqrt{20 + 4y}}{2} = \frac{-2 \pm \sqrt{4(5+y)}}{2} = \frac{-2 \pm 2\sqrt{5+y}}{2}$

$= -1 \pm \sqrt{5+y}$

Since $x \leq -1$, we have to use the $-$

$f^{-1}(y) = -1 - \sqrt{5+y}$