Algebra and Trigonometry — Exam 5 MAT 150, Fall 2017 — D. Ivanšić	Name: Saul Ocean *Show all your work!*

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ $\qquad$ $\sin(2u) = 2\sin u \cos u$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ $\qquad$ $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

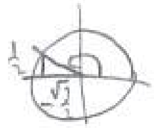
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ $\qquad$ $\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$

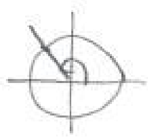
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\qquad$ $\sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ $\qquad$ $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

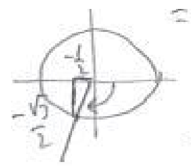
1. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\cos 150° = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ $\qquad$ $\csc(-120°) = \frac{1}{\sin(-120°)}$ $\qquad$ $\tan \frac{5\pi}{2} = \frac{y}{x} = \frac{1}{0}$

not defined

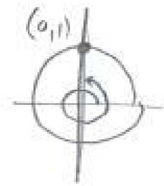




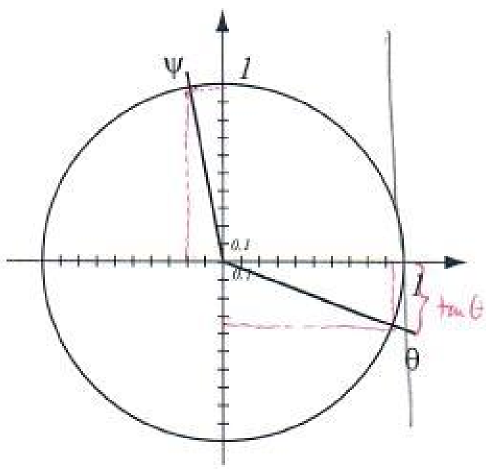


$= \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$



2. (8pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn. Note the angles are **not** the standard angles.



$\sin\theta = -0.35$ $\qquad$ $\tan\theta = -0.4 \left(\text{or } \frac{-0.35}{0.95} \approx -0.37\right)$

$\cos\psi = -0.2$ $\qquad$ $\csc\psi = \frac{1}{\sin\psi} = \frac{1}{0.97} = 1.03$

3. (8pts) Use identities to simplify the following expression.

$\sin\left(\frac{\pi}{2} - \theta\right)\cos\theta - \cos\left(\frac{\pi}{2} - \theta\right)\sin(-\theta) =$

$= \cos\theta \cos\theta - \sin\theta\,(-\sin\theta)$

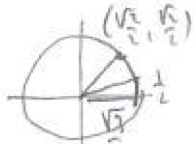
$= \cos^2\theta + \sin^2\theta = 1$

4. (14pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$$\cos\frac{5\pi}{12}^{\circ} = \cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

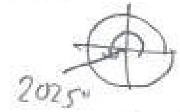
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$



$\sin 202.5^{\circ} = \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$

$\hookrightarrow \frac{405^{\circ}}{2}$

$360^{\circ}+45^{\circ}$ ✓



$-$ since 202.5° is in Q3

$$\sin^2\frac{405^{\circ}}{2} = \frac{1-\cos 405^{\circ}}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2} = \frac{2-\sqrt{2}}{4}$$

$\sin 202.5^{\circ} = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$

Show the identities:

5. (8pts) $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta} = 2\sec\theta\tan\theta$

$$\frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta} = \frac{1+\sin\theta-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2\sin\theta}{\cos^2\theta} = 2\cdot\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} = 2\sec\theta\tan\theta$$

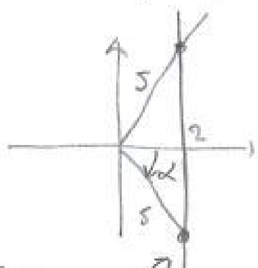
6. (8pts) $\dfrac{\sin\theta\cos\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)} = \dfrac{1}{2}\tan(2\theta)$

$$\frac{\sin\theta\cos\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)} = \frac{\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta} = \frac{\sin\theta\cos\theta}{\cos(2\theta)}\cdot\frac{2}{2} = \frac{\overbrace{2\sin\theta\cos\theta}^{\sin(2\theta)}}{2\cos(2\theta)}$$

$$= \frac{\sin(2\theta)}{\cos(2\theta)}\cdot\frac{1}{2} = \frac{1}{2}\tan(2\theta)$$

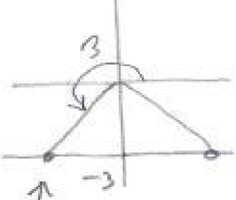
7. (12pts) Suppose that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\pi < \beta < \frac{3\pi}{2}$ are angles so that $\cos\alpha = \frac{2}{5}$ and $\sin\beta = -\frac{3}{4}$. Using identities (sum, difference, half- or double-angle) and without using the calculator, find the exact value of $\tan(\alpha + \beta)$.

$\cos\alpha = \frac{2}{5} = \frac{x}{r}$ $\quad 2^2 + y^2 = 5^2$ $\quad y^2 = 21$ $\quad y = \pm\sqrt{21}$ $\quad y = -\sqrt{21}$ $\quad \tan\alpha = -\frac{\sqrt{21}}{2}$



$\frac{3\pi}{2} \le \alpha < 2\pi$

$\sin\beta = \frac{-3}{4} = \frac{y}{r}$ $\quad x^2 + (-3)^2 = 4^2$ $\quad x^2 = 7$ $\quad x = \pm\sqrt{7}$ $\quad x = -\sqrt{7}$ $\quad \tan\beta = \frac{3}{\sqrt{7}}$



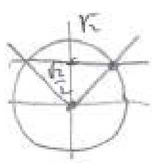
$\pi < \beta < \frac{3\pi}{2}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-\frac{\sqrt{21}}{2} + \frac{3}{\sqrt{7}}}{1 - \left(-\frac{\sqrt{21}}{2}\cdot\frac{3}{\sqrt{7}}\right)} \cdot \frac{2\sqrt{7}}{2\sqrt{7}} = \frac{-\sqrt{21}\sqrt{7} + 6}{2\sqrt{7} + 3\sqrt{21}} = \boxed{\frac{6 - 7\sqrt{3}}{2\sqrt{7} + 3\sqrt{21}}}$$

8. (10pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

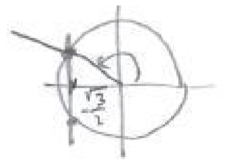
$\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\qquad \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ $\qquad \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\qquad \arcsin 2 =$ not defined

$\cos\theta = \frac{\sqrt{2}}{2}$

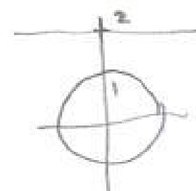


$\tan\theta = -\frac{1}{\sqrt{3}}$

$\cos\theta = -\frac{\sqrt{3}}{2}$



$\sin\theta = 2$



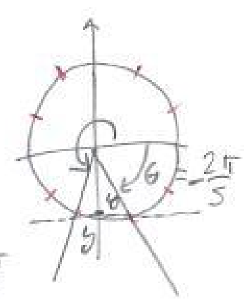
no intersection, so not def.

9. (8pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

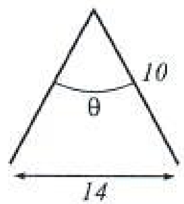
$\cos(\arccos 0.3) = 0.3$ $\qquad \arcsin\left(\sin\frac{7\pi}{5}\right) = \arcsin y = -\frac{2\pi}{5}$

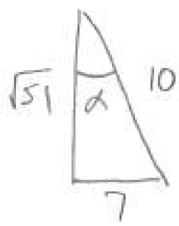
$y = \sin\theta$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



10. (12pts) A 10-ft folding ladder is placed on a floor so that its ends are 14 feet apart. Find the exact value for $\sin\theta$ (do not use the calculator), where θ is the angle the ladder subtends.





$b^2+7^2=10^2$

$b^2=51$

$b=\sqrt{51}$

$\sin\alpha=\frac{7}{10}$

$\cos\alpha=\frac{\sqrt{51}}{10}$

$\theta=2\alpha$

$\sin\theta=\sin(2\alpha)=2\sin\alpha\cos\alpha=2\cdot\frac{7}{\cancel{10}_{5}}\cdot\frac{\sqrt{51}}{10}=\boxed{\frac{7\sqrt{51}}{50}}$

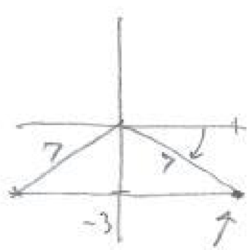
Bonus. (10pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$\cos\left(2\arcsin\left(-\frac{3}{7}\right)\right)=\cos 2\theta=\cos^2\theta-\sin^2\theta$

Let $\theta=\arcsin(-\frac{3}{7})$

$=\left(\frac{2\sqrt{10}}{7}\right)^2-\left(-\frac{3}{7}\right)^2=\frac{40}{49}-\frac{9}{49}=\boxed{\frac{31}{49}}$

$\sin\theta=-\frac{3}{7}$



$x^2+(-3)^2=7^2$

$x^2=40$

$x=\pm\sqrt{40}$

$x=2\sqrt{10}$

$\cos\theta=\frac{2\sqrt{10}}{7}$

Since θ is in $[-\frac{\pi}{2},\frac{\pi}{2}]$

being $\arcsin(-\frac{3}{7})$