Algebra and Trigonometry — Exam 4
MAT 150, Fall 2017 — D. Ivanšić

Name: Saul Ocean

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1. (5pts) If $\log_a 2 = u$ and $\log_a 3 = v$, express in terms of u and v:

$\log_a 6 =$ $\log_a(2\cdot 3)$
$= \log_a 2 + \log_a 3$
$= u + v$

$\log_a \frac{3}{8} = \log_a \frac{3}{2^3} = \log_a 3 - \log_a 2^3$
$= \log_a 3 - 3\log_a 2$
$= v - 3u$

2. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \frac{\sqrt[5]{u^4}}{49w^3} = \log_7 u^{\frac{4}{5}} - \log_7 49 - \log_7 w^3$
$= \frac{4}{5}\log_7 u - 2 - 3\log_7 w$

3. (6pts) Write as a single logarithm. Simplify if possible.

$2\ln(x^3y^{-2}) - 3\ln(x^{-2}y^5) = \ln(x^3y^{-2})^2 - \ln(x^{-2}y^5)^3 = \ln \frac{(x^3y^{-2})^2}{(x^{-2}y^5)^3}$

$6-(-6)=12$, $-4-15=-19$

$= \ln \frac{x^6y^{-4}}{x^{-6}y^{15}} = \ln(x^{12}y^{-19}) = \ln \frac{x^{12}}{y^{19}}$

Solve the equations.

4. (6pts) $9^{2x-7} = \left(\frac{1}{3}\right)^{2x+6}$

$(3^2)^{2x-7} = (3^{-1})^{2x+6}$
$3^{4x-14} = 3^{-2x-6}$
$4x-14 = -2x-6$
$6x = 8$
$x = \frac{8}{6} = \frac{4}{3}$

5. (8pts) $5^{2x+1} = 4^{-3x+4}$

$\ln$
$\ln 5^{2x+1} = \ln 4^{-3x+4}$
$(2x+1)\ln 5 = (-3x+4)\ln 4$
$2x\ln 5 + \ln 5 = -3x\ln 4 + 4\ln 4$
$2x\ln 5 + 3x\ln 4 = 4\ln 4 - \ln 5$
$x(2\ln 5 + 3\ln 4) = 4\ln 4 - \ln 5$
$x = \frac{4\ln 4 - \ln 5}{2\ln 5 + 3\ln 4} = 0.53346$

6. (12pts) The population of Expandaton was 127,000 in 2012 and 154,000 in 2017. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2012. Graph it on paper.
b) Find the predicted population in the year 2021.

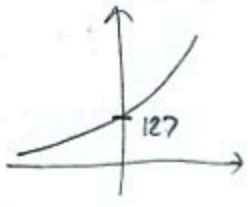
a) $P(t) = 127\,e^{kt}$ (in thousands)

$154 = 127\,e^{k\cdot 5}$

$\frac{154}{127} = e^{k\cdot 5}$ | ln

$\ln\frac{154}{127} = k\cdot 5$

$k = \frac{\ln\frac{154}{127}}{5} = 0.0385531$



b) In 2021, $t = 9$

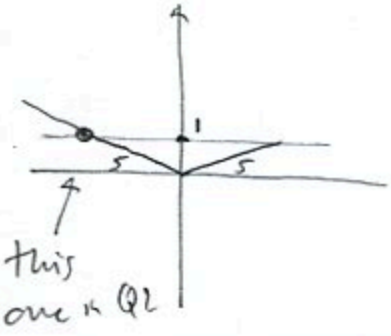
$P(9) = 127\,e^{0.0385531\cdot 9}$

$P(9) = 179.677758$

About 179,678 people

7. (10pts) If $\sin\theta = \frac{1}{5}$ and θ is in the second quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

$\sin\theta = \frac{1}{5} = \frac{y}{r}$



$x^2 + 1^2 = 5^2$

$x^2 = 24$

$x = \pm 2\sqrt{6}$

$x = -2\sqrt{6}$

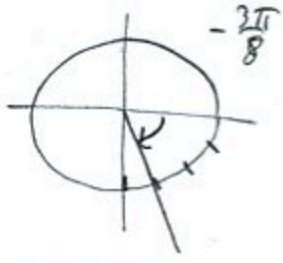
$\sin\theta = \frac{1}{5}$ $\quad \csc\theta = 5$

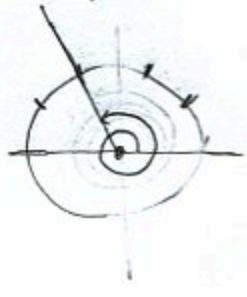
$\cos\theta = -\frac{2\sqrt{6}}{5}$ $\quad \sec\theta = -\frac{5}{2\sqrt{6}}$

$\tan\theta = -\frac{1}{2\sqrt{6}}$ $\quad \cot\theta = -2\sqrt{6}$

8. (6pts) Sketch angles in standard position with indicated radian measure.

$\frac{7\pi}{6}$ $\qquad -\frac{3\pi}{8}$ $\qquad \frac{13\pi}{5\,8} = 2\frac{3}{5}\pi$



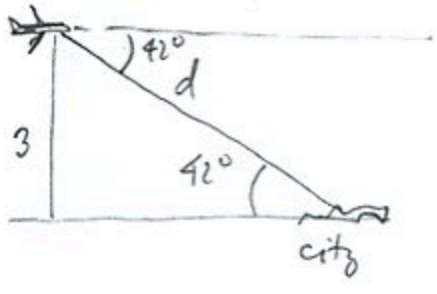


9. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$9° = 9 \cdot \frac{\pi}{180} = \frac{\pi}{20}$ radians $= 0.15708$ radians

$\frac{7\pi}{15}$ radians $= \frac{7\pi}{15} \cdot \frac{180}{\pi} = 84°$

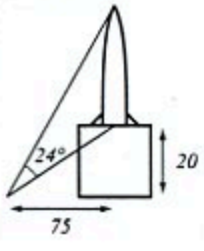
10. (9pts) An airplane is flying at altitude 3 miles when it spots a city in the distance. If the angle of depression to the city is 42°, what is the line-of-sight (through the air) distance from the airplane to the city?



$\frac{3}{d} = \sin 42°$

$d = \frac{3}{\sin 42°} = 4.48343$ miles

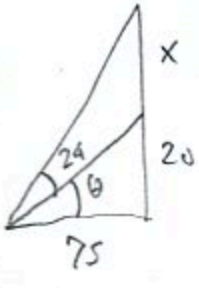
11. (14pts) From a point on the ground 75 meters away from the launch pad, you observe a rocket and note it subtends an angle of 24°. If the launch pad is 20 meters tall, how tall is the rocket?



$\tan\theta = \frac{20}{75} = \frac{4}{15}$

$\theta = \tan^{-1}\frac{4}{15}$

$= 14.931417°$



$\frac{x+20}{75} = \tan(\theta+24°)$

$x+20 = 75\tan(\theta+24°)$

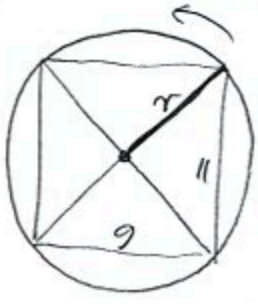
$x = 75\tan(38.931417°) - 20$

$x = 40.585302$ meters

12. (12pts) Our textbook has dimensions approximately 9×11 inches. Skillful student Fred is rotating it on his finger at 50 revolutions per minute. (His finger touches the center of the book.)

a) What is the angular velocity of this rotation in radians per second?

b) What is the linear velocity of the corner of the book, in inches per second?



a) $\omega = \frac{\theta}{t} = \frac{2\pi \cdot 50 \text{ radians}}{60 \text{ sec}} = \frac{5\pi}{3}$ radians/sec. $= 5.235988$ rad/sec

b) $v = r\omega = \frac{\sqrt{202}}{2} \cdot \frac{5\pi}{3} = \frac{5\pi\sqrt{202}}{6}$ in/sec $= 37.208684$ in/sec

$(2r)^2 = 9^2 + 11^2$

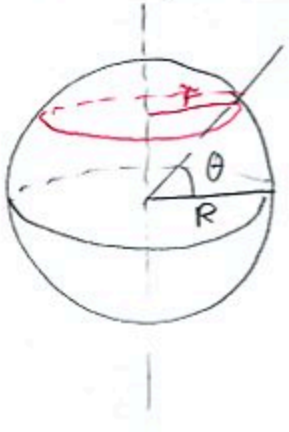
$4r^2 = 81 + 121$

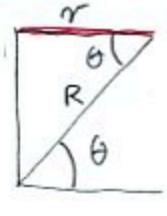
$r^2 = \frac{202}{4}$

$r = \pm\frac{\sqrt{202}}{2}$

$r = \frac{\sqrt{202}}{2} = 7.106335$

Bonus. (10pts) Let θ be the latitude of a point on Earth, R the radius of the Earth. The point circles the axis of Earth's rotation on a circle of a radius r (the farther north the point, the smaller the r). Write the formula for r using R and θ.





$\frac{r}{R} = \cos\theta$

$r = R\cos\theta$