Algebra and Trigonometry — Exam 3
MAT 150, Fall 2017 — D. Ivanšić

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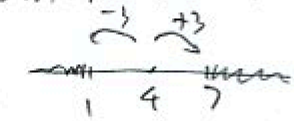
Show all your work!

1. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x-4| > 3$

dist. from x to $4 > 3$

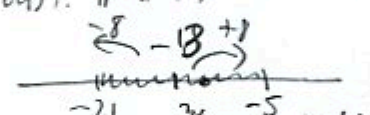


$(-\infty, 1) \cup (7, \infty)$

$|3x+13| \le 8$

$|3x-(-13)| \le 8$

dist. from $3x$ to $-13 \le 8$



$\div 3$

$[-7, -\frac{5}{3}]$

Solve the equations:

2. (8pts) $\dfrac{7x-51}{x^2+9x+14} + \dfrac{2x-6}{x+7} = \dfrac{x}{x+2}$ $\Big| \cdot (x+2)(x+7)$

$(x+7)(x+2)$

$7x-51+(2x-6)(x+2) = x(x+7)$

$7x-51+2x^2-2x-12 = x^2+7x \quad |-x^2-7x$

$x^2-2x-63=0$

$(x-9)(x+7)=0$

$x=9$ or -7

↑ gives 0 in denom.

$\boxed{\text{only } x=9 \text{ is sol.}}$

3. (8pts) $x+\sqrt{51+10x} = -6$

$\sqrt{51+10x} = -x-6$

$\sqrt{51+10x} = -(x+6) \quad |^2$

$51+10x = \underbrace{(-1)^2}_{=1}(x^2+12x+36)$

$x^2+12x+36 = 51+10x \quad |-10x-51$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5, 3$

$\boxed{\text{no solution}}$

Check:

$-5+\sqrt{51-50} \overset{?}{=} -6$

$-5+1 \overset{?}{=} -6$ no

$3+\sqrt{51+30} \overset{?}{=} -6$

$3+9 \overset{?}{=} -6$ no

4. (8pts) Evaluate without using the calculator. Show how you got the numbers.

$\log_8 64 = 2$ $\qquad 8^? = 64$

$\log_3 \frac{1}{27} = -3$ $\qquad 3^? = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$\log_8 16 = \frac{4}{3}$ $\qquad 8^? = 16 = 2^4 = (\sqrt[3]{8})^4 = 8^{\frac{4}{3}}$

$\log_{\sqrt[3]{b}} b^2 = 6$ $\qquad (\sqrt[3]{b})^? = b^2 \quad (b^{\frac{1}{3}})^? = b^2$

5. (4pts) Use the change-of-base formula and your calculator to find $\log_{11} 27$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_{11} 27 = \dfrac{\log 27}{\log 11} = 1.374471$

6. (3pts) For the polynomial $P(x) = 4x^5 - 8x^3 + 4x^2 - 7x + 1$ state:

degree: 5 leading coefficient: 4 leading term: $4x^5$

7. (14pts) The polynomial $f(x) = (x-5)^2(x+3)(x+1)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

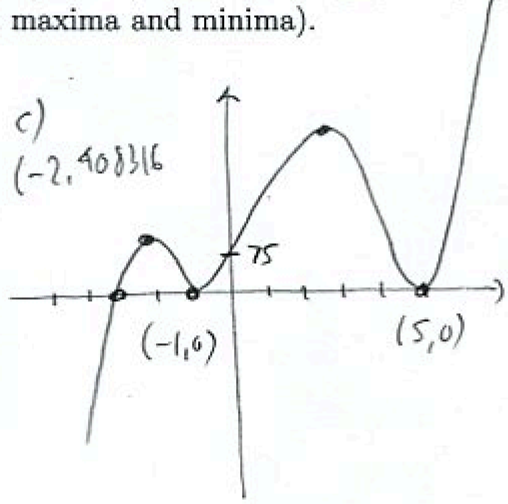
a) $x^2 \cdot x \cdot x^2 = x^5$
like x^5

b)

zero	5	-3	-1
mult.	2	1	2

y-int: $f(0) = (-5)^2 \cdot 3 \cdot 1^2$
$= 75$

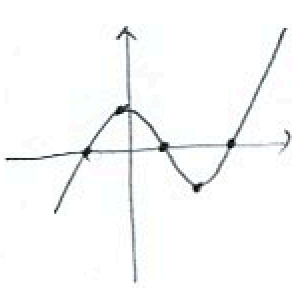
c)
(-2.408316



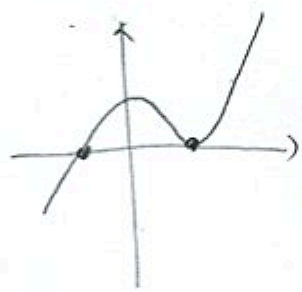
d) Turning points: $(-2.408316, 64.406404)$
$(-1, 0)$
$(2.408316, 421.9936)$
$(5, 0)$

8. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of x-intercepts and turning points.
b) Draw a graph of P that has exactly 2 x-intercepts.
c) Draw a graph of P that has no turning points.

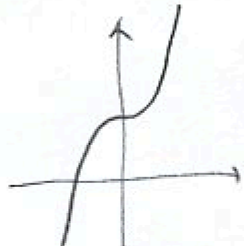
a)



b)



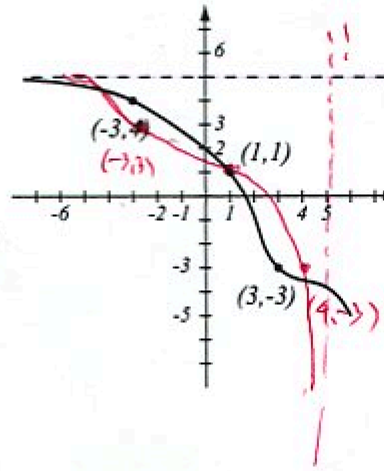
c)



e.g., $y = x^3 + 4$

9. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.

a) Yes - passes horizontal line test



10. (9pts) Let $f(x) = \dfrac{5}{x^3 - 2}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \dfrac{5}{x^3 - 2}$

$y(x^3 - 2) = 5$

$x^3 - 2 = \dfrac{5}{y}$

$x^3 = \dfrac{5}{y} + 2$

$x = \sqrt[3]{\dfrac{5}{y} + 2}$

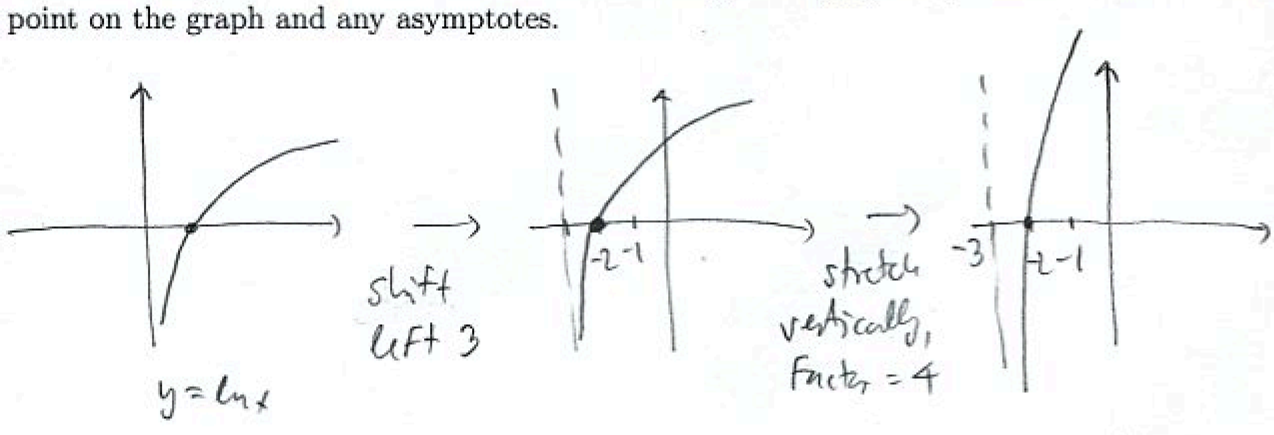
$f^{-1}(y) = \sqrt[3]{\dfrac{5}{y} + 2}$

b) Range f = Domain of f^{-1}

Can't have: $y = 0$

Range of f: $(-\infty, 0) \cup (0, \infty)$

11. (6pts) Using transformations, draw the graph of $f(x) = 4\ln(x+3)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



12. (8pts) Find the domain of the function $f(x) = \dfrac{\log_5(3x+8)}{|x+4|-2}$ and write it in interval notation.

Must have:

$3x+8>0$

$3x>-8$

$x>-\frac{8}{3}$

Can't have

$|x+4|-2=0$

$|x+4|=2$

$x+4=2$ or $x+4=-2$

$x=-2$ or $x=-6$

$-6 \quad -\frac{8}{3} \quad -2$

$(-\frac{8}{3},-2)\cup(-2,\infty)$

13. (8pts) How much should you invest in an account bearing 2.3%, compounded daily, if you wish to have \$2,500 in five years?

$A=P(1+\frac{r}{n})^{nt}$

$2500=P\left(1+\frac{0.023}{365}\right)^{365\cdot 5}$

$2500=P\cdot 1.000063014^{1825}$

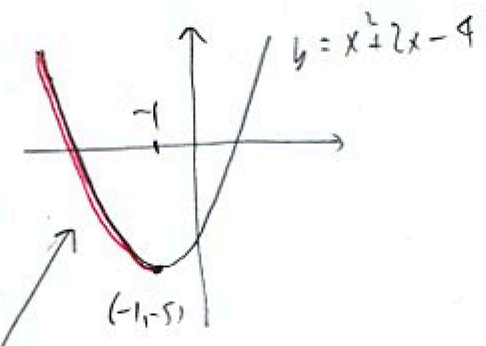
$2500=P\cdot 1.121869$

$P=\dfrac{2500}{1.121869}=2228.42$

Bonus. (10pts) Let $f(x)=x^2+2x-4$ with domain $x\leq -1$.
a) Sketch the graph of the function. Is it a one-to-one function?
b) Find $f^{-1}(x)$. *(Hint: quadratic formula.)*

a) $h=-\frac{b}{2a}=-\frac{2}{2\cdot 1}=-1$

$k=(-1)^2+2\cdot(-1)-4=-5$



red: graph of our function

It is one-to-one, passes horizontal line test,

b) $y=x^2+2x-4$

$x^2+2x-4-y=0$ solve for x

$x=\dfrac{-2\pm\sqrt{2^2-4\cdot 1\cdot(-4-y)}}{2\cdot 1}$

$=\dfrac{-2\pm\sqrt{4+4(y+4)}}{2}=\dfrac{-2\pm\sqrt{4y+20}}{2}$ $\quad 4(y+5)$

$=\dfrac{-2\pm 2\sqrt{y+5}}{2}=-1\pm\sqrt{y+5}$

Because $x\leq -1$, we take the $-$ sign,

so $f^{-1}(y)=-1-\sqrt{y+5}$