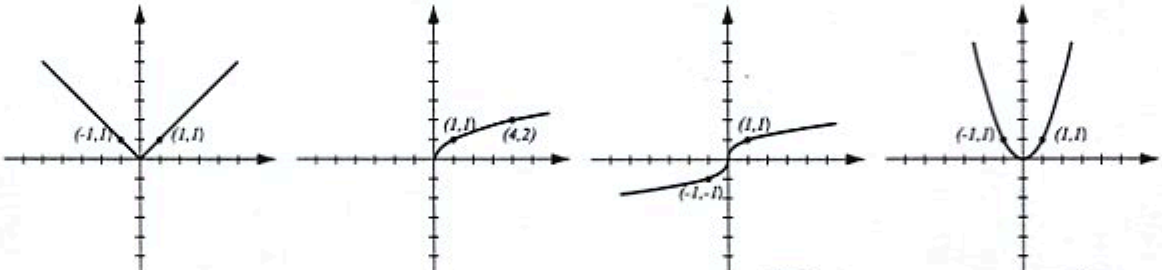
Algebra and Trigonometry — Exam 2 **MAT 150, Fall 2017 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $\quad$ $y = \sqrt{x}$ $\quad$ $y = \sqrt[3]{x}$ $\quad$ $y = x^2$

Simplify, so that the answer is in form $a + bi$.

2. (4pts) $\dfrac{3+7i}{2i} = \dfrac{3+7i}{2i} \cdot \dfrac{(-2i)}{(-2i)} = \dfrac{-6i - 14i^2}{-4\underbrace{i^2}_{=-1}} = \dfrac{14-6i}{4} = \dfrac{7}{2} - \dfrac{3}{2}i$

3. (4pts) Simplify and justify your answer.

$i^{86} = i^{84} \cdot i^{2} = \underbrace{(i^4)}_{=1}{}^{21} \cdot i^2 = i^2 = -1$

4. (17pts) Let $f(x) = \dfrac{x-1}{x^2-4}$, $g(x) = \sqrt{x} - 3$.

Find the following (simplify where possible):

$(fg)(4) = f(4)g(4) = \dfrac{3}{12} \cdot (\sqrt{4} - 3)$

$= \dfrac{1}{4} \cdot (-1) = -\dfrac{1}{4}$

$(g \circ f)(1) = g(f(1)) = g\left(\dfrac{0}{-4}\right) = g(0) = -3$

$\dfrac{g}{f}(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x}-3}{\dfrac{x-1}{x^2-4}} = \sqrt{x} - 3 \cdot \dfrac{x^2-4}{x-1}$

$= \dfrac{(x^2-4)(\sqrt{x}-3)}{x-1}$

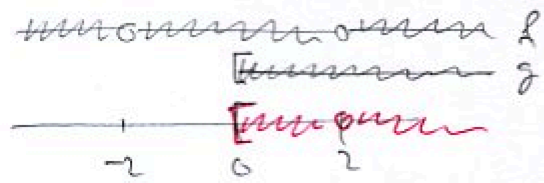
$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}-3) = \dfrac{\sqrt{x}-3-1}{(\sqrt{x}-3)^2-4}$

$= \dfrac{\sqrt{x}-4}{\sqrt{x}^2 - 6\sqrt{x} + 9 - 4} = \dfrac{\sqrt{x}-4}{x - 6\sqrt{x} + 5}$

The domain of $f - g$ in interval notation

Domain of f: Can't have $x^2 - 4 = 0$ $\quad$ $x^2 = 4$ $\quad$ $x = \pm 2$

Domain of g: Must have $x \geq 0$



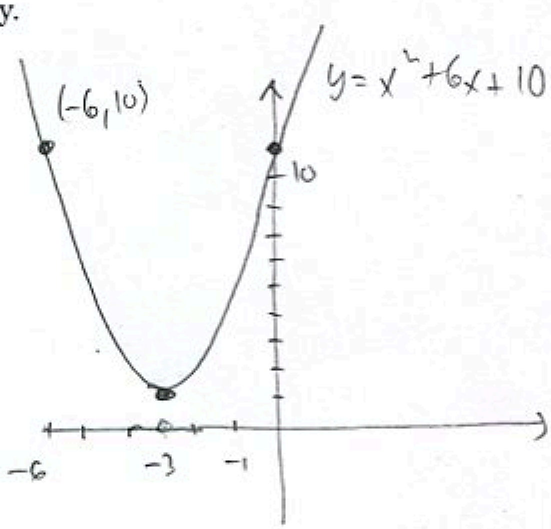
Domain of $f - g$ is $[0, 2) \cup (2, \infty)$

5. (12pts) The quadratic function $f(x) = x^2 + 6x + 10$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0) = 10$

x-int: $x^2+6x+10=0$

$$x = \frac{-6 \pm \sqrt{6^2-4\cdot1\cdot10}}{2\cdot1} = \frac{-6\pm\sqrt{-4}}{2}$$

no real solutions, so no x-int.

b) $h = -\frac{b}{2a} = -\frac{6}{2\cdot1} = -3$

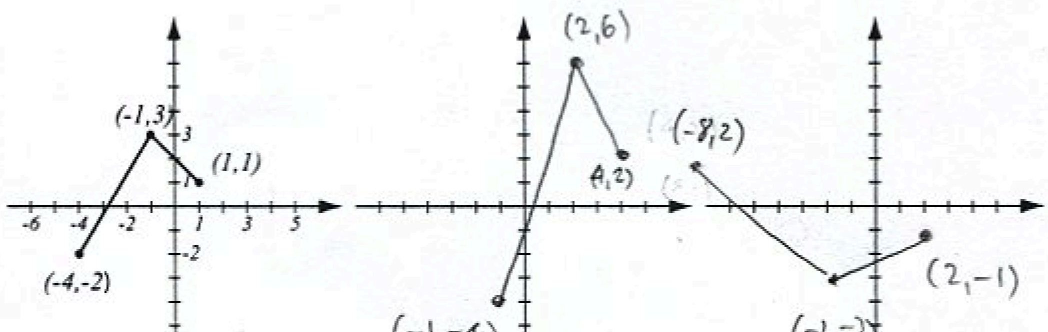
$k = f(-3) = (-3)^2+6\cdot(-3)+10 = 1$

6. (6pts) Consider the function $h(x) = \sqrt{x^2 - 3x + 5}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2-3x+5$ $\qquad$ $g(x) = x^2-3x$

$f(x) = \sqrt{x}$ $\qquad$ $f(x) = \sqrt{x+5}$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-3)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



shift right 3
vertical stretch, factor 2

horizontal stretch by factor 2
reflect in x-axis

8. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 20 = 0 \quad | + 4^2 - 20$

$x^2 - 2 \cdot x \cdot 4 + 4^2 = 16 - 20$

$(x-4)^2 = -4$

$x - 4 = \pm 2i$

$x = 4 \pm 2i$

9. (7pts) For the function $f(x) = x^4 - 6x^2 + 2$:
a) determine algebraically whether it is odd, even, or neither
b) use the calculator to draw its graph here and verify your conclusion by stating symmetry.

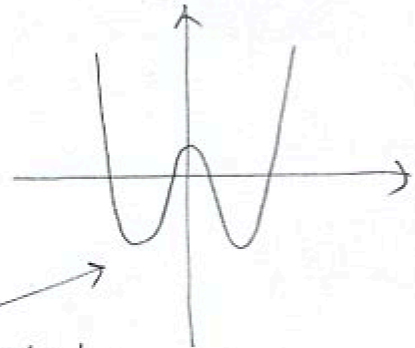
$f(-x) = (-x)^4 - 6(-x)^2 + 2$

$= x^4 - 6x^2 + 2$

$= f(x)$

So, even function

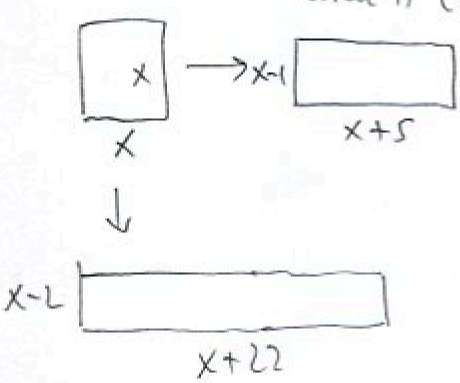
symmetric about y-axis



indeed

10. (12pts) Starting with a square, we decrease the width by 1 inch and increase the length by 5 inches to arrive at a rectangle. Starting with the same square, we decrease the width by 2 inches and increase the length by 22 inches to arrive at another rectangle whose area is twice the area of the earlier rectangle. How long is the side of the square?

area is $(x-1)(x+5)$



area is $(x-2)(x+22)$

$2(x-1)(x+5) = (x-2)(x+22)$

$2(x^2+4x-5) = x^2+20x-44$

$2x^2+8x-10 = x^2+20x-44$

$x^2 - 12x + 34 = 0$ 144−136

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 1 \cdot 34}}{2 \cdot 1} = \dfrac{12 \pm \sqrt{8}}{2}$

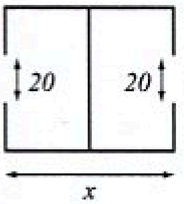
$= \dfrac{12 \pm 2\sqrt{2}}{2} = 6 \pm \sqrt{2}$

Since both $6 \pm \sqrt{2}$ are greater than 2, both are solutions

11. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 800 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



$A = xy = x\left(280 - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + 280x$

$800 = 2x + 2(y-20) + y$

$800 = 2x + 3y - 40$

$3y = 840 - 2x$

$y = \frac{840-2x}{3} = 280 - \frac{2}{3}x$

Domain: $x \geq 0$

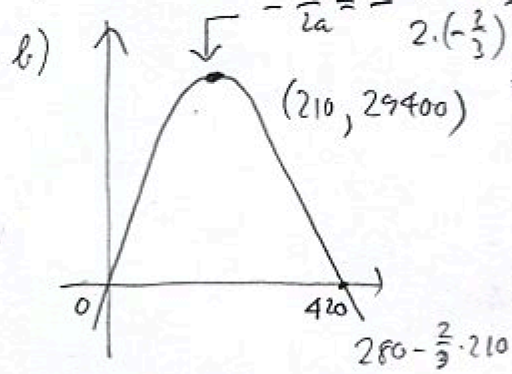
Must have $y \geq 20$ $\quad [0, 390]$

$280 - \frac{2}{3}x \geq 20$

$260 \geq \frac{2}{3}x$

$x \leq 260 \cdot \frac{3}{2} = 390$

b) $-\frac{b}{2a} = -\frac{280}{2\cdot\left(-\frac{2}{3}\right)} = \frac{3\cdot 280}{4} = 210$

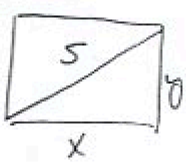


$280 - \frac{2}{3}\cdot 210$

Max area for 210×140

Max area is 29400 square feet.

Bonus. (10pts) Among all rectangles with diagonal of length 5 cm, find the dimensions of the one with the greatest perimeter. What is the greatest possible perimeter?



$P = 2x + 2y = 2x + 2\sqrt{25-x^2}$

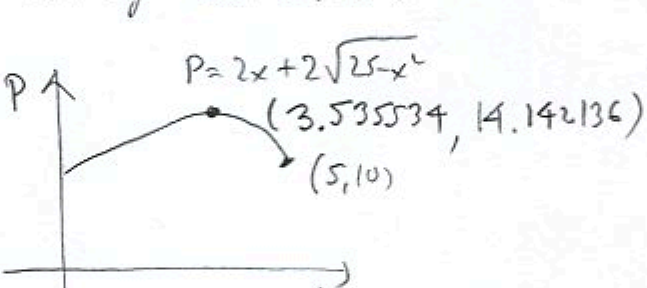
$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25-x^2}$

$= \sqrt{25-x^2}$

since $y \geq 0$



Max area occurs for rectangle 3.535534×3.535534

and it is 14.142136 cm