

1. (8pts) Let  $A = (-3, -2)$  and  $B = (5, 0)$ .

a) Find the midpoint  $M$  of  $A$  and  $B$ .

b) Verify that the distances from  $M$  to  $A$  and  $M$  to  $B$  are equal.

2. (10pts) Write the equation of the circle with center  $(-1, -2)$  that contains the point  $(2, 5)$ . Sketch the circle.

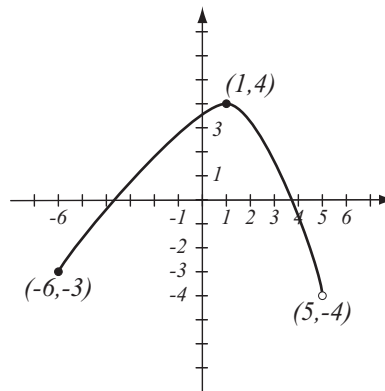
3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

a) Find  $f(1)$  and  $f(5)$ .

b) What is the domain of  $f$ ?

c) What is the range of  $f$ ?

d) What are the solutions of the equation  $f(x) = -2$ ?



4. (12pts) The function  $f(x) = x^4 + 5x^3 + 4x^2 - 7x - 18$  is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt[3]{4x - 11}}{7x - 5}$$

$$g(x) = \frac{\sqrt{x}}{x^2 - 2x - 15}$$

6. (10pts) Let  $g(x) = \frac{2x - 4}{x^2 - 1}$ . Find the following (simplify where appropriate).

$$g(3) =$$

$$g(1) =$$

$$g(-z) =$$

$$g(w + 3) =$$



5. (12pts) A business that manufactures decorative plates keeps track of its expenses. One month, it produced 124 plates and had expenses of \$2977.76. Another month, it produced 185 plates and had expenses of \$4029.40.

a) Assuming that the business expenses  $E(x)$  is a linear function of the number of plates  $x$  produced, write a formula for  $E(x)$ .

b) What are the expenses if no plates are produced? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A farmer is trying to establish the relationship between the amount of rainfall during growing season and the yield of corn on his farm. The table shows the data:  $W$  is the amount of rainfall in millimeters and  $Y$  is the corn yield in tons. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between  $W$  and  $Y$ . Draw the line on the graph.

c) Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.

d) Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $W$  and  $Y$ ?

e) What yield can the farmer expect if the the amount of rainfall in a year is 750 millimeters?

| $W$ | $Y$  |
|-----|------|
| 535 | 10.2 |
| 556 | 10.7 |
| 625 | 11   |
| 705 | 11.3 |
| 790 | 11.5 |
| 810 | 11.9 |
| 888 | 12.5 |

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $2 \leq 5 - 3x < 11$

2. (7pts)  $2x + 7 < 5$  or  $5x - 2 > 13$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{9 - 2x}}{2x - 6}$ .

4. (14pts) Two computer-maintenance freelancers charge for their services in this way: George charges \$150 for the first three hours and then \$40 per hour for hours after the first three; Yuri charges \$200 for the first five hours and then \$45 per hour for hours after the first five. Assuming at least five hours of work are needed, for which number of hours is it preferable to hire George? Solve as an inequality.

5. (14pts) A 183-mile-long road joins cities Charlestown and Crown City. At the same time, one tractor leaves Charlestown and drives toward Crown City, and another tractor, driving 3mph faster than the first tractor, leaves Crown City and drives toward Charlestown. After 2 hours they meet on the road.

- a) What are the speeds of the tractors?
- b) How far from Charlestown did they meet?

6. (14pts) How many liters of a 7% solution of sulphuric acid must be mixed with 3 liters of an 12% solution of sulphuric acid in order to get a 9% solution of sulphuric acid?

1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{3x^2 + 7x - 2}{x^2 + 2}$ . Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = \sqrt{x+4}$ ,  $g(x) = \frac{x-3}{x^2-6}$ . Find the following (simplify where possible):

$$(f+g)(-4) =$$

$$(fg)(5) =$$

$$\frac{f}{g}(x) =$$

$$(f \circ g)(2) =$$

$$(g \circ f)(x) =$$

The domain of  $(fg)(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = \sqrt[3]{4x^2 + 7}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

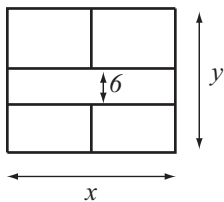
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 3, & \text{if } -4 \leq x < 1 \\ x - 2, & \text{if } x \geq 1. \end{cases}$$

5. (14pts) A builder is charged with designing a simple school house with area 6000 square feet, four rooms and a corridor at least 6 feet wide. The school board wishes to minimize the cost, which is the same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the school house for which the total length of the walls is minimal? What is the minimal wall length?





1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^5 - 7x^3 + 4x$$

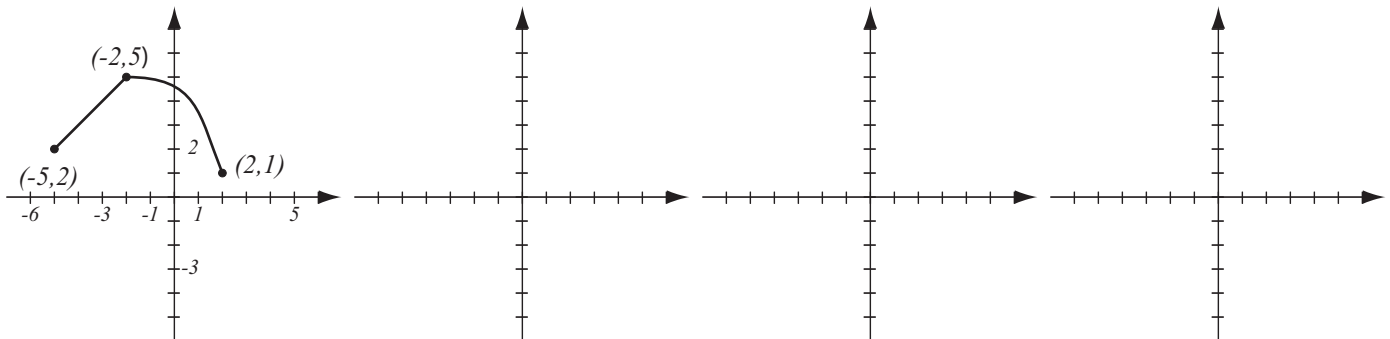
$$g(x) = x^2 - 5|x| - 7$$

$$h(x) = x^3 - 3x^2 + 2$$

2. (16pts) Using transformations, draw the graphs of  $f(x) = -\sqrt[3]{x+4}$  and  $g(x) = 5 + \sqrt{-2x}$ . Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = |x|$ , shifted up 2 units
  - shape of  $y = \frac{1}{x}$  stretched vertically by factor 3, then shifted right 4 units
  - shape of  $y = x^3$ , stretched horizontally by factor 4, then reflected about the  $y$ -axis, then shifted left 1 unit.

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(x) - 2$ ,  $-f(2x)$  and  $2f(x - 3)$  and label all the relevant points.



College Algebra — Joysheet 6  
MAT 140, Fall 2017 — D. Ivanišić

Name: \_\_\_\_\_

*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $4 - 3i + (3 - i) \cdot 4i =$

2. (6pts)  $\frac{5 + 7i}{4 - 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{213} =$

4. (8pts) The amount of food (in tons) arriving daily to a hurricane-stricken area is given by  $A(x) = -x^2 + 20x + 15$ , where  $x$  is the number of days after September 20th. On what dates were 106 tons arriving daily?

5. (8pts) Solve the equation:  $x^4 + 3x^2 - 40 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 14x + 42 = 0$

7. (12pts) The quadratic function  $f(x) = 4x^2 - 4x - 15$  is given. Do the following without using the calculator.

- a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (12pts) Greg is in charge of irrigating a large field and his weekly pay depends on the weekly rainfall. His base pay of \$10 an hour is supplemented by hardship hourly pay of \$1 per inch of rainfall (for working in a muddy field). However, if it rains, less work on irrigation is needed, so his 40 weekly hours are reduced by 2 hours per inch of rainfall. One week, Greg was paid \$442. How many inches of rain fell during that week?

1. (4pts) Solve the equation.

$$|3x - 1| = 7$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x - 5| \geq 2$$

$$|2x + 9| < 3$$

Solve the equations:

3. (8pts)  $\frac{x + 4}{x + 3} + \frac{x^2 - 6x + 43}{x^2 + x - 6} - \frac{x + 5}{x - 2} = 0$

4. (8pts)  $x + 3 = 2x + \sqrt{29 - 5x}$

5. (14pts) A ball is thrown upwards from the ground with initial velocity 21 meters per second. Its height in meters after  $t$  seconds is given by  $s(t) = -5t^2 + 21t$ .

a) Sketch the graph of the height function.

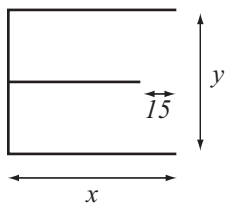
b) When does the ball reach its greatest height, and what is that height?

c) When is the ball at height 22 meters?

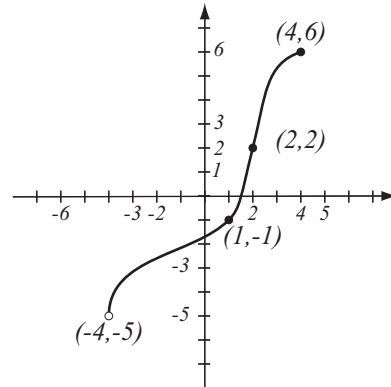
6. (14pts) Truck mechanic Igor wishes to build a repair shop with two side-by-side bays separated by a shorter wall (see picture). Igor has enough money to build 1200 feet of walls, and he wants to build a shop with maximal area.

a) Express the total area of the shop as a function of one of the sides of the rectangle. What is the domain of this function?

b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the greatest total area? What is the greatest area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $f(x) = \frac{3x + 1}{2x - 5}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator:

$$\log_4 64 =$$

$$\log_2 \frac{1}{32} =$$

$$\log_{125} 25 =$$

$$\log_{\sqrt{b}} b^3 =$$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_7 0.27$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$2,000 and annual interest rate of 3.24%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

| Frequency: every | $n$ | Amount after 1 year |
|------------------|-----|---------------------|
| year             |     |                     |
| quarter          |     |                     |
| month            |     |                     |
| day              |     |                     |
| hour             |     |                     |
| second           |     |                     |

6. (3pts) Find the domain of  $f(x) = \log_3(4x + 9)$ .

7. (8pts) An object weighing 500 kilograms moving at velocity  $v$  meters per second has kinetic energy  $E(v) = 250v^2$  Joules.

- Determine the kinetic energy of the object if it is traveling at 10 and 20 meters per second.
- Find a formula for the inverse function and explain what it represents.
- Determine the velocity of the object if its kinetic energy is 60,000 and 120,000 Joules.

8. (7pts) Using transformations, draw the graph of  $f(x) = 4 - e^x$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.



1. (5pts) If  $\log_a 2 = 0.235409$  and  $\log_a 7 = 0.660876$ , find (show how you obtained your numbers):

$$\log_a \frac{7}{2} =$$

$$\log_a 56 =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 (81x^2y^6) =$$

$$\log_5 \frac{x^{\frac{3}{4}}y^3}{625\sqrt{x}\sqrt[3]{y^7}} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$\frac{1}{3} \log(27y^{12}) - 4 \log(2y^{\frac{3}{8}}) - 4 \log x =$$

$$3 \log_4(x+9) - 2 \log_4(x^2+5x-36) - \log_4(x-4) =$$

Solve the equations.

4. (5pts)  $16^{3x-1} = \left(\frac{1}{8}\right)^{x+3}$

5. (7pts)  $7^{x+5} = 6^{4x-1}$

6. (8pts)  $\log_2(x-1) + \log_2(x+3) = 5$

7. (12pts) According to US census data, Lexington, KY had 260,512 inhabitants in 2000 and 295,803 in 2010. Assume the population of Lexington grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Lexington  $t$  years after 2000. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 400,000?