College Algebra — Joysheet 9 MAT 140, Fall 2017 — D. Ivanšić	Name: Saul Ocean *Covers: 5.4–5.6* *Show all your work!*

1. (5pts) If $\log_a 2 = 0.235409$ and $\log_a 7 = 0.660876$, find (show how you obtained your numbers):

$\log_a \frac{7}{2} = \log_a 7 - \log_a 2$
$= 0.660876 - 0.235409$
$= 0.425467$

$\log_a 56 = \log_a 8 \cdot 7 = \log_a 2^3 \cdot 7$
$= \log_a 2^3 + \log_a 7$
$= 3\log_a 2 + \log_a 7$
$= 3 \cdot 0.235409 + 0.660876 = 1.367103$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3(81x^2y^6) = \log_3 81 + \log_3 x^2 + \log_3 y^6$
$= 4 + 2\log_3 x + 6\log_3 y$

$\log_5 \frac{x^{\frac{3}{4}}y^3}{625\sqrt{x}\sqrt[3]{y^7}} = \log_5 x^{\frac{3}{4}} + \log_5 y^3 - \log_5 625 - \log_5 x^{\frac{1}{2}} - \log_5 y^{\frac{7}{3}}$

$x^{\frac{1}{2}}$ $x^{\frac{2}{3}}$

$= \frac{3}{4}\log_5 x + 3\log_5 y - 4 - \frac{1}{2}\log_5 x - \frac{7}{3}\log_5 y$
$= \frac{1}{4}\log_5 x + \frac{2}{3}\log_5 y - 4$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\log(27y^{12}) - 4\log(2y^{\frac{3}{8}}) - 4\log x = \log(27y^{12})^{\frac{1}{3}} - \log(2y^{\frac{3}{8}})^4 - \log x^4$
$= \log(27^{\frac{1}{3}} y^{12\cdot\frac{1}{3}}) - \log(2^4 y^{\frac{3}{8}\cdot 4}) - \log x^4$
$= \log(3y^4) - \log(16y^{\frac{3}{2}}) - \log x^4 = \log\frac{3y^4}{16y^{\frac{3}{2}}x^4} = \log\frac{3y^{\frac{5}{2}}}{16x^4}$

$3\log_4(x+9) - 2\log_4(x^2+5x-36) - \log_4(x-4) =$

$= \log_4(x+9)^3 - \log_4(\underbrace{x^2+5x-36}_{(x+9)(x-4)})^2 - \log_4(x-4)$

$= \log_4\frac{(x+9)^3}{((x+9)(x-4))^2(x-4)} = \log_4\frac{(x+9)^{\cancel{3}\,1}}{(x+9)^{\cancel{2}\,0}(x-4)^3} = \log_4\frac{x+9}{(x-4)^3}$

Solve the equations.

4. (5pts) $16^{3x-1} = \left(\frac{1}{8}\right)^{x+3}$

$(2^4)^{3x-1} = (2^{-3})^{x+3}$

$2^{12x-4} = 2^{-3x-9}$

$12x - 4 = -3x - 9$

$15x = -5$

$x = -\frac{5}{15} = -\frac{1}{3}$

5. (7pts) $7^{x+5} = 6^{4x-1}$ | ln

$\ln 7^{x+5} = \ln 6^{4x-1}$

$(x+5)\ln 7 = (4x-1)\ln 6$

$x\ln 7 + 5\ln 7 = 4x\ln 6 - \ln 6$

$x\ln 7 - 4x\ln 6 = -5\ln 7 - \ln 6$

$x(\ln 7 - 4\ln 6) = -5\ln 7 - \ln 6$

$x = \frac{-5\ln 7 - \ln 6}{\ln 7 - 4\ln 6} = \frac{5\ln 7 + \ln 6}{4\ln 6 - \ln 7}$

$= 2.206671$

6. (8pts) $\log_2(x-1) + \log_2(x+3) = 5$

$\log_2((x-1)(x+3)) = 5$ | $2^{\wedge}$

$2^{\log_2((x-1)(x+3))} = 2^5$

$(x-1)(x+3) = 32$

$x^2 + 2x - 3 = 32$

$x^2 + 2x - 35 = 0$

$(x-5)(x+7) = 0$

$x = 5, -7$

-7 gives negative in a log, so not a sol.

$\boxed{x = 5}$

7. (12pts) According to US census data, Lexington, KY had 260,512 inhabitants in 2000 and 295,803 in 2010. Assume the population of Lexington grows exponentially.
a) Write the function describing the number $P(t)$ of people in Lexington t years after 2000. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 400,000?

a) $P(t) = 260{,}512\,e^{kt}$

time	Population
2000 $(t=0)$	260,512
2010 $(t=10)$	295,803

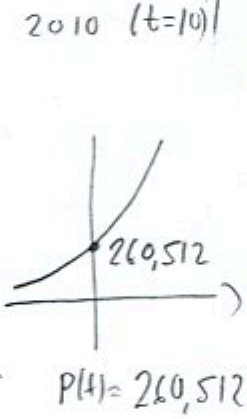
$P(10) = 295{,}803$

$260{,}512\,e^{k\cdot 10} = 295{,}803$

$e^{10k} = \frac{295{,}803}{260{,}512}$ | ln

$10k = \ln\frac{295{,}803}{260{,}512}$

$k = \frac{\ln\frac{295{,}803}{260{,}512}}{10} = 0.0127045$



$P(t) = 260{,}512\,e^{0.0127045t}$

c) $P(t) = 400{,}000$

$260{,}512\,e^{0.012..t} = 400{,}000$

$e^{0.012..t} = \frac{400000}{260{,}512}$ | ln

$0.012..t = \ln\frac{400{,}000}{260{,}512}$

$t = \frac{\ln\frac{400{,}000}{260{,}512}}{0.0127045} = 33.753112$

About 34 years from 2000, so, in 2034.