College Algebra — Joysheet 4
MAT 140, Fall 2017 — D. Ivanšić

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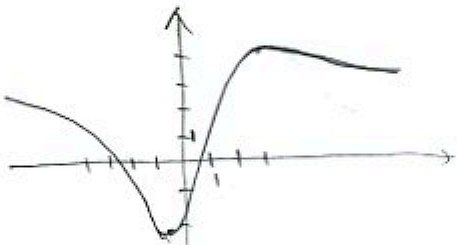
Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{3x^2+7x-2}{x^2+2}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



$y = \dfrac{3x^2+7x-2}{x^2+2}$

a) $f(-0.675419) = -2.181981$ is a local min

$f(2.961131) = 4.181981$ is a local max

b) Increasing on $(-0.675419, 2.961131)$

Decreasing on $(-\infty, -0.675419)$ and $(2.961131, \infty)$

2. (20pts) Let $f(x) = \sqrt{x+4}$, $g(x) = \dfrac{x-3}{x^2-6}$. Find the following (simplify where possible):

$(f+g)(-4) = f(-4)+g(-4)$

$= \sqrt{-4+4} + \dfrac{-4-3}{16-6} = 0 + \dfrac{-7}{10} = -\dfrac{7}{10}$

$(fg)(5) = f(5)g(5) = \sqrt{5+4} \cdot \dfrac{5-3}{25-6}$

$= 3 \cdot \dfrac{2}{19} = \dfrac{6}{19}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x+4}}{\frac{x-3}{x^2-6}} = \sqrt{x+4} \cdot \dfrac{x^2-6}{x-3}$

$= \dfrac{(x^2-6)\sqrt{x+4}}{x-3}$

$(f\circ g)(2) = f(g(2)) = f\left(\dfrac{2-3}{4-6}\right) = f\left(\dfrac{-1}{-2}\right)$

$= f\left(\dfrac{1}{2}\right) = \sqrt{\dfrac{1}{2}+4} = \sqrt{\dfrac{9}{2}} = \dfrac{3}{\sqrt{2}}$

$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x+4}\right) = \dfrac{\sqrt{x+4}-3}{\sqrt{x+4}^2-6} =$

$= \dfrac{\sqrt{x+4}-3}{x+4-6} = \dfrac{\sqrt{x+4}-3}{x-2}$

The domain of $(fg)(x)$ in interval notation

Domain of f: Must have $x+4 \geq 0$, $x \geq -4$

Domain of g: Can't have $x^2-6=0$, $x^2=6$, $x = \pm\sqrt{6}$

$x \neq \pm\sqrt{6}$

Domain of fg: $[-4,-\sqrt{6}) \cup (-\sqrt{6},\sqrt{6}) \cup (\sqrt{6},\infty)$

3. (8pts) Consider the function $h(x) = \sqrt[3]{4x^2+7}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 4x^2+7$ $\qquad$ $g(x) = 4x^2$

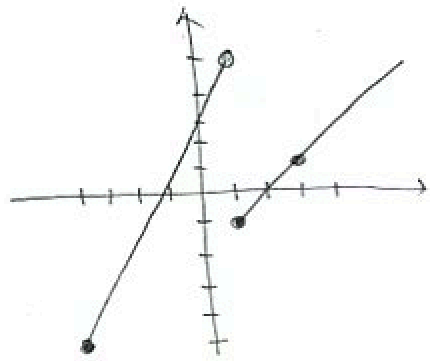
$f(x) = \sqrt[3]{x}$ $\qquad$ $f(x) = \sqrt[3]{x+7}$

4. (8pts) Sketch the graph of the piecewise-defined function:

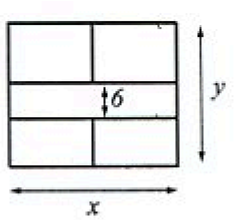
$$f(x) = \begin{cases} 2x+3, & \text{if } -4 \le x < 1 \\ x-2, & \text{if } x \ge 1. \end{cases}$$

x	$2x+3$
-4	-5
1	5

x	$x-2$
1	-1
3	1



5. (14pts) A builder is charged with designing a simple school house with area 6000 square feet, four rooms and a corridor at least 6 feet wide. The school board wishes to minimize the cost, which is the same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the school house for which the total length of the walls is minimal? What is the minimal wall length?



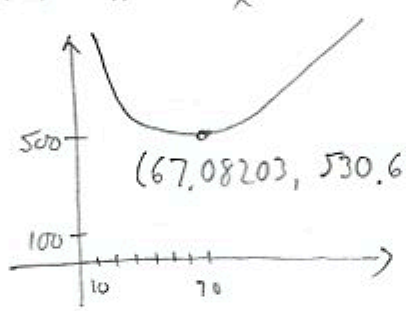
$l = 4x + 2y + y - 6$

$= 4x + 3y - 6$

$= 4x + 3 \cdot \frac{6000}{x} - 6 =$

$l(x) = 4x + \frac{18000}{x} - 6$

$A = 6000 = xy$ so $y = \frac{6000}{x}$



Domain: $x > 0$

$y \ge 6$

$\frac{6000}{x} \ge 6$

$\frac{6000}{6} \ge x$

$x \le 1000$

$(0, 1000]$

$\frac{6000}{67.08203}$

Dimensions of school house:

$67.08203 \times 89.442732$

Minimal wall length $= 530.65631$ ft