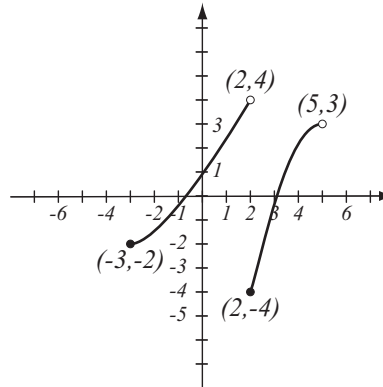


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(-3) =$ $f(2) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 1$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 3x^2 - 5x - 8$.

- Draw the graph on paper and indicate units on the axes.
- Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (5pts) Write the equation of the line whose x -intercept is 2 and passes through (7, 4).

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $4x - 3y = 6$ and passes through the y -intercept of the given line. Draw both lines.

5. (7pts) Draw the line $y = \frac{2}{3}x + 2$. This line and the x - and y -axes determine a triangle. Find the perimeter of this triangle.

6. (9pts) Let $f(x) = \frac{2x - 5}{x^2 - 4x}$. Find the following (simplify where appropriate).

$$g(4) =$$

$$g(6) =$$

$$g(-3x) =$$

$$g(u + 1) =$$

7. (10pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{4}{x^2 + 2x - 15}$$

$$g(x) = \frac{\sqrt{2x + 5}}{2x - 5}$$

8. (5pts) Solve the inequality and write your solution in interval notation.

$$-2 \leq 3x + 1 \leq 9$$

9. (10pts) The endpoints of a diameter of a circle are $(-3, 4)$ and $(1, 2)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) Linda has these options for a data plan for her cell phone:

A) \$18 flat fee for the first two GB, and then \$7 per GB for usage beyond the first two GB.

B) \$8 per GB.

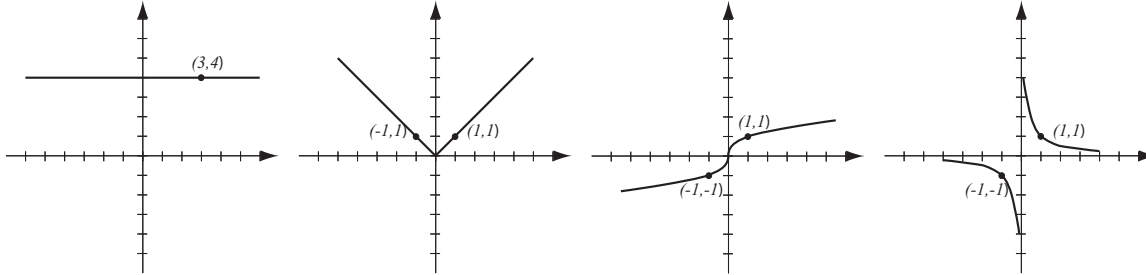
Assuming Linda always uses at least 2 GB of data, for which amount of data is plan B better?

11. (14pts) Pablo drives to a job interview in an hour and a half. Returning along the same route, he feels more relaxed and drives 11mph slower, so it takes him an hour and three quarters.

- a) How fast is Pablo driving on the way to and from the job interview?
- b) How far did he travel one-way?

Bonus (10pts) Betty has a total of \$4000 invested in two accounts, one bearing 6% and the other 7% interest. She notices that if she reversed the amounts invested in each account, she would have \$16 more in interest over a year. How much is invested in each account?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (21pts) Let $f(x) = 2x + 1$, $g(x) = \frac{x - 5}{x^2 - 4}$.

Find the following (simplify where possible):

$$(f + g)(1) =$$

$$(fg)(3) =$$

$$\frac{f}{g}(x) =$$

$$(f \circ g)(0) =$$

$$(g \circ f)(x) =$$

The domain of $\frac{g}{f}$ in interval notation

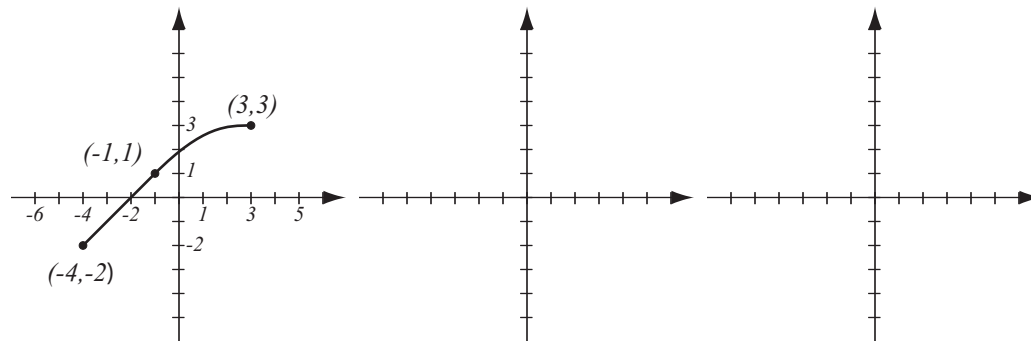
3. (6pts) Consider the function $h(x) = \frac{7}{x^2 + 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:

a) shape of $y = x^2$, shifted down 2 units

b) shape of $y = \sqrt[3]{x}$, stretched horizontally by factor $\frac{1}{4}$, then reflected over the x -axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x - 1)$ and $f(-x) + 4$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x, & \text{if } x \leq -1 \\ 3x - 4, & \text{if } -1 < x \leq 4 \end{cases}$$

7. (7pts) For the function $f(x) = x^5 - 7x^3 + 4$:

a) Determine algebraically whether it is odd, even, or neither.

b) Use the calculator to draw its graph here and verify your conclusion by stating symmetry.

8. (20pts) Let $f(x) = x^4 - 3x^2 - 4$ (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.

b) Determine algebraically whether the function is odd, even, or neither.

c) Verify your conclusion from b) by stating symmetry.

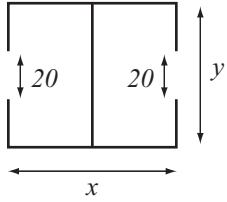
d) Find the local maxima and minima for this function.

e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. The warehouse is to have area 40,000 square feet and the company's goal is to minimize the total length of the walls.

a) Express the total wall length as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the warehouse that has the smallest total wall length and what is the minimal wall length?



Bonus. (10pts) Let $f(x) = x^2 + 2x - 4$ and $g(x) = \sqrt{x + 5} - 1$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

College Algebra — Exam 3
MAT 140, Fall 2017 — D. Ivanišić

Name: _____
Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(2 - 3i)^2 - 3 - 5i =$

2. (5pts) $\frac{3 + 7i}{1 + 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{175} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 20 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x + 9| \geq 2$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.

- Draw a graph of P that has the maximal number of x -intercepts and turning points.
- Draw a graph of P that has exactly 3 x -intercepts.
- Can the graph of P have no turning points? Justify.

7. (12pts) The quadratic function $f(x) = x^2 - 6x - 16$ is given. Do the following without using the calculator.

- a) Find the x - and y -intercepts of its graph, if any.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

Solve the equations:

8. (8pts) $\frac{x+4}{x-6} + \frac{x^2-15x-36}{x^2-3x-18} = \frac{x+1}{x+3}$

9. (8pts) $2\sqrt{x+11} - 3 = x$

10. (14pts) The polynomial $f(x) = \frac{1}{10}(x + 5)^2(x - 7)^2$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y -intercept.

c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).

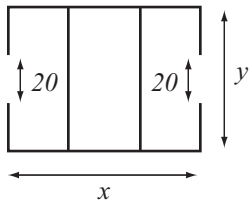
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a rectangle, length is 6 inches more than the width. If we increase the width by 12 inches and decrease the length by 5 inches we arrive at a rectangle whose area is twice the area of the original rectangle. What are the dimensions of the original rectangle?

12. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1000 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Find \sqrt{i} , that is, find all complex numbers $x + yi$ so that $(x + yi)^2 = i$. To solve this equation, expand the left side, and solve for x and y using the fact that real and imaginary parts of both sides must be equal. Keep in mind that both x and y are real numbers.

College Algebra — Exam 4
MAT 140, Fall 2017 — D. Ivanišić

Name: _____
Show all your work!

1. (8pts) Evaluate without using the calculator:

$$\log_4 256 =$$

$$\log_3 \frac{1}{9} =$$

$$\log_a \sqrt[3]{a^5} =$$

$$\log_{b^2} b^{10} =$$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_3 10$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 3 = u$ and $\log_a 7 = v$, express in terms of u and v :

$$\log_a 49 =$$

$$\log_a \frac{\sqrt{3}}{7} =$$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_5 \frac{125y^3}{\sqrt[5]{x^6}} =$$

5. (6pts) Write as a single logarithm. Simplify if possible.

$$2 \log_7(x^{-4}y^4) - 3 \log_7(x^2y^{-3}) =$$

6. (4pts) Simplify.

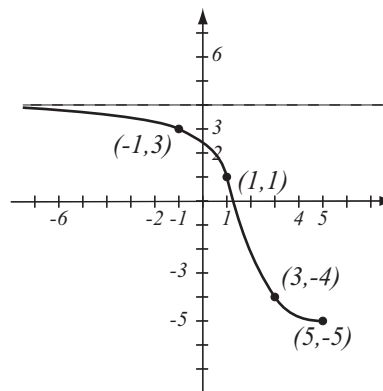
$$\log_8 8^{\sqrt{5}} =$$

$$e^{\ln(3-x^2)} =$$

7. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points, and showing any asymptotes.



8. (8pts) Let $f(x) = 4 + \sqrt{x + 3}$.

a) Find the formula for f^{-1} .

b) Find the range of f^{-1} .

9. (6pts) Using transformations, draw the graph of $f(x) = -\ln(x + 3)$. Explain how you transform the graph of a basic function in order to get the graph of f . Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \log_{11}(3 - 5x)$ and write it in interval notation.

11. (8pts) How much should you invest in an account bearing 2.4%, compounded monthly, if you wish to have \$2,000 in four years?

Solve the equations.

Solve the equations.

12. (6pts) $25^{x+1} = \left(\frac{1}{5}\right)^{2x+4}$

13. (8pts) $3^{2x+1} = 4^{9-x}$

14. (10pts) $\log_3(x - 8) + \log_3(2x + 5) = 4$

15. (12pts) The population of Splodaton was 35,000 in 2009 and 42,000 in 2014. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 2009. Graph it on paper.

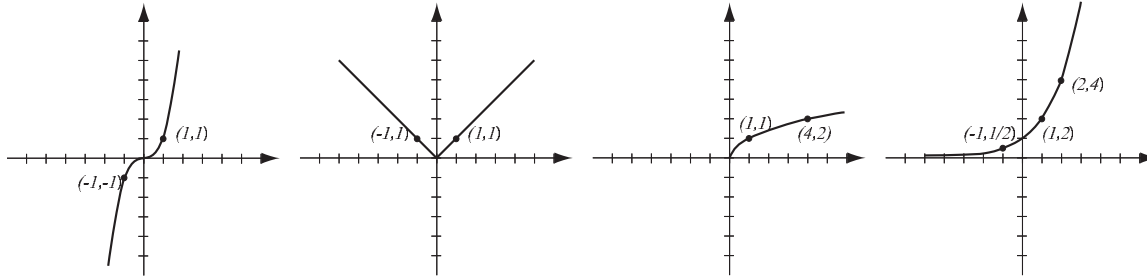
b) Find the predicted population in the year 2019.

Bonus (10pts) Let $f(x) = \frac{e^x - 1}{e^x + 1}$.

a) Graph the function (sketch on paper!). Explain why it is one-to-one.

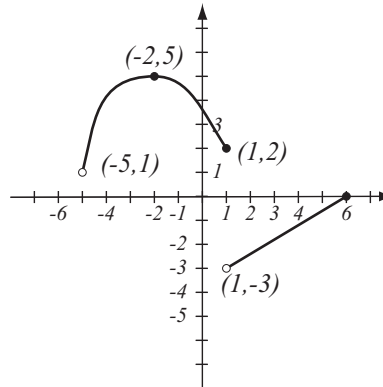
b) Find the formula for $f^{-1}(x)$.

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(-2) =$ $f(1) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 2$?



3. (9pts) Write the equation of the line whose x -intercept is -3 and passes through $(1, 2)$. Is this line perpendicular to the line $x + 2y = 7$? Draw both lines.

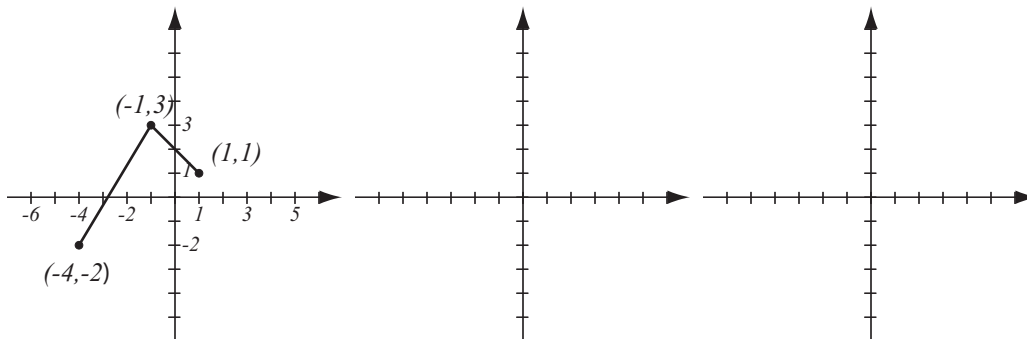
4. (6pts) Solve the inequality. Write the solution in interval form.

$$|2x - 3| < 4$$

5. (6pts) Find the domain of the function $f(x) = \frac{\ln(5 - 2x)}{x^2 - 3x - 18}$ and write it in interval notation.

6. (5pts) Let $f(x) = 4 + (x - 5)^3$. Find the formula for f^{-1} .

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x - 3)$ and $f\left(\frac{1}{2}x\right) + 1$ and label all the relevant points.



8. (12pts) The quadratic function $f(x) = x^2 - 2x + 7$ is given. Do the following without using the calculator.

a) Find the x - and y -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \frac{x^2}{81 \sqrt[4]{y^7}} =$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$\log(x^3 y^{-5}) - 4 \log(xy^{-2}) =$$

11. (20pts) The polynomial $P(x) = x^4 - 13x^2 + 36$ is given (answer with 6 decimals accuracy).

- a) What is the end behavior of the polynomial?
- b) Factor the polynomial to find all the zeros and their multiplicities. Find the y -intercept.
- c) Determine algebraically whether the function is odd, even, or neither.
- d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
- e) Verify your conclusion from c) by stating symmetry.
- f) Find all the turning points (i.e., local maxima and minima).

Solve the equations.

12. (8pts) $x + \sqrt{4x + 17} = 1$

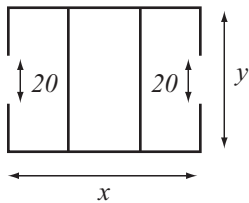
13. (8pts) $3^{2x+1} = 4^x$

14. (14pts) Pablo drives to a job interview in an hour. Returning along the same route, he feels more relaxed and drives 14mph slower, so it takes him an hour and a quarter.

- How fast is Pablo driving on the way to and from the job interview?
- How far did he travel one-way?

15. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1400 feet of walls, and its goal is to maximize the total area of the warehouse.

- Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
- Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



16. (12pts) The population of Splodaton was 24,000 in 2011 and 30,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 2011. Graph it on paper.

b) Find the predicted population in the year 2022.

Bonus (10pts) Betty has a total of \$7000 invested in two accounts, one bearing 5% and the other 8% interest. She notices that if she reversed the amounts invested in each account, she would have \$78 more in interest over a year. How much is invested in each account?