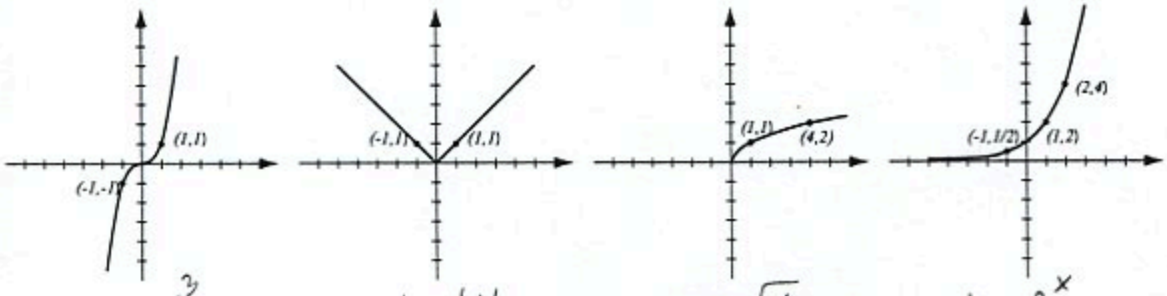
College Algebra — Final Exam
MAT 140, Fall 2017 — D. Ivanšić

Name: Saul Ocean

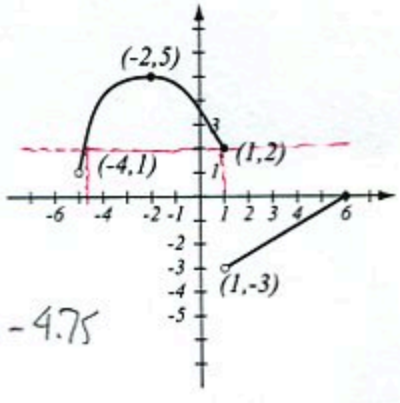
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = x^3$ $y = |x|$ $y = \sqrt{x}$ $y = 2^x$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-2) = 5$ $f(1) = 2$

b) What is the domain of f? $(-5, 6]$

c) What is the range of f? $(-3, 0] \cup (1, 5]$

d) What are the solutions of the equation $f(x) = 2$? $x = 1$ and $x = -4.75$

3. (9pts) Write the equation of the line whose x-intercept is -3 and passes through $(1, 2)$. Is this line perpendicular to the line $x + 2y = 7$? Draw both lines.

Line goes through $(-3, 0)$ and $(1, 2)$

$m = \frac{2-0}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - (-3))$

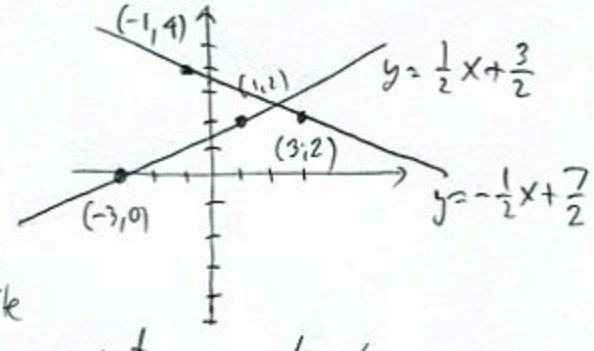
$y = \frac{1}{2}x + \frac{3}{2}$

$x + 2y = 7$

$2y = -x + 7$

$y = -\frac{1}{2}x + \frac{7}{2}$

slopes are not opposite reciprocal, so lines are not perpendicular

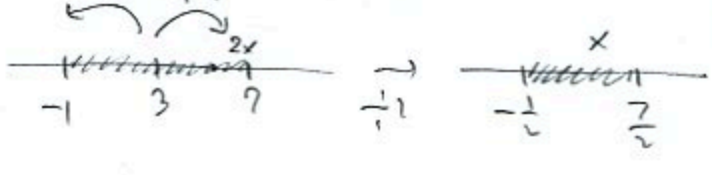


4. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 3| < 4$

distance from $2x$ to $3 < 4$

Sol: $(-\frac{1}{2}, \frac{7}{2})$



5. (6pts) Find the domain of the function $f(x) = \dfrac{\ln(5-2x)}{x^2-3x-18}$ and write it in interval notation.

Must have: $5-2x>0$

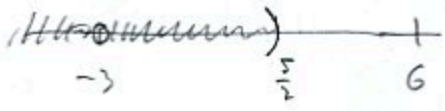
$5>2x$

$\frac{5}{2}>x$

Can't have: $x^2-3x-18=0$

$(x-6)(x+3)=0$

$x=6,-3$



$(-\infty,-3)\cup(-3,\frac{5}{2})$

6. (5pts) Let $f(x) = 4+(x-5)^3$. Find the formula for f^{-1}.

$y = 4+(x-5)^3$

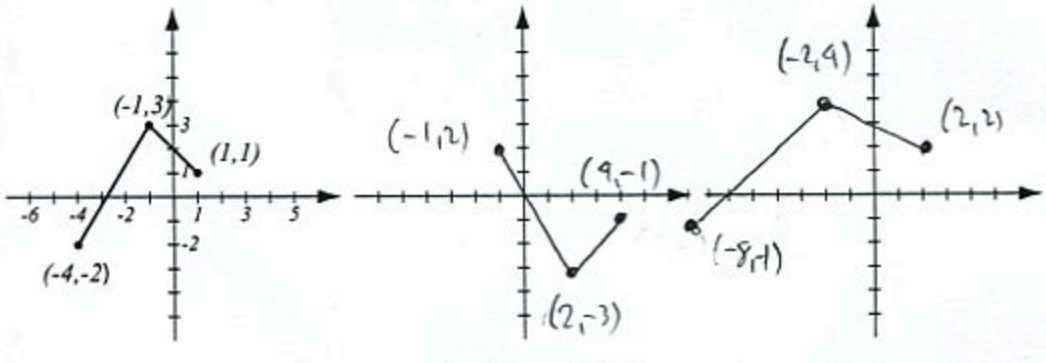
$y-4=(x-5)^3$

$\sqrt[3]{y-4} = x-5$

$x = \sqrt[3]{y-4}+5$

$f^{-1}(y) = \sqrt[3]{y-4}+5$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-3)$ and $f\left(\frac{1}{2}x\right)+1$ and label all the relevant points.



shift right 3
reflect in x-axis

stretch horizontally, factor = 2
shift up 1

8. (12pts) The quadratic function $f(x) = x^2 - 2x + 7$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

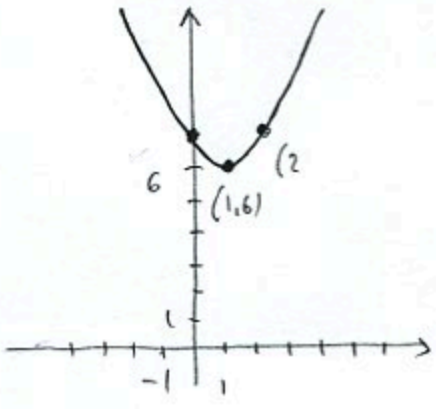
c) Sketch the graph of the function.

a) $x^2 - 2x + 7 = 0$ $\quad 4 - 28$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{-24}}{2}$$ not real, so no x-int

y-int $= f(0) = 7$



b) Vertex: $h = -\frac{-2}{2 \cdot 1} = 1$

$k = f(1) = 1 - 2 + 7 = 6$

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \frac{x^2}{81\sqrt[4]{x^7}} = \log_3 x^2 - \log_3 81 - \log_3 x^{\frac{7}{4}}$$

$$= 2\log_3 x - 4 - \frac{7}{4}\log_3 x$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$\log(x^3y^{-5}) - 4\log(xy^{-2}) = \log(x^3y^{-5}) - \log(xy^{-2})^4$$

$$= \log \frac{x^3y^{-5}}{(xy^{-2})^4} = \log \frac{x^3y^{-5}}{x^4y^{-8}} = \log x^{-1}y^3 = \log \frac{y^3}{x}$$

11. (20pts) The polynomial $P(x) = x^4 - 13x^2 + 36$ is given (answer with 6 decimals accuracy).

a) What is the end behavior of the polynomial?

b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.

c) Determine algebraically whether the function is odd, even, or neither.

d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).

e) Verify your conclusion from c) by stating symmetry.

f) Find all the turning points (i.e., local maxima and minima).

a) like x^4 

b) Let $u = x^2$. Then

$x^4 - 13x^2 + 36 = u^2 - 13u + 36$

$= (u-4)(u-9) = (x^2-4)(x^2-9)$

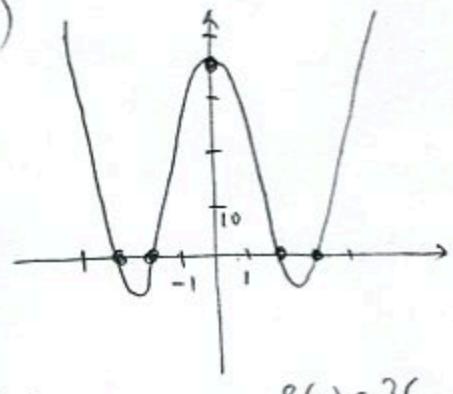
$= (x-2)(x+2)(x-3)(x+3)$

zero	-2	2	-3	3
mult.	1	1	1	1

y-int: $P(0) = 36$

c) $P(-x) = (-x)^4 - 13(-x)^2 + 36$

$= x^4 - 13x^2 + 36 = P(x)$ so even.

d) 

f) Local max is $f(0) = 36$

Local min are: $f(-2.549508) = -6.25$

$f(2.549508) = 6.25$

Solve the equations.

12. (8pts) $x + \sqrt{4x+17} = 1$

$\sqrt{4x+17} = 1 - x \quad |^2$

$4x + 17 = 1 - 2x + x^2 \quad |-4x-17$

$x^2 - 6x - 16 = 0$

$(x+2)(x-8) = 0$ $\boxed{x=-2}$

$x = -2, 8$

Check: $-2 + \sqrt{4(-2)+17} \stackrel{?}{=} 1$

$-2 + \sqrt{9} = 1$ yes

$8 + \sqrt{4 \cdot 8 + 17} = 1$

$8 + \sqrt{49} = 1$ no

13. (8pts) $3^{2x+1} = 4^x \quad | \ln$

$\ln 3^{2x+1} = \ln 4^x$

$(2x+1)\ln 3 = x \ln 4$

$2x \ln 3 + \ln 3 = x \ln 4$

$2x \ln 3 - x \ln 4 = -\ln 3$

$x(2\ln 3 - \ln 4) = -\ln 3$

$x = \dfrac{-\ln 3}{2\ln 3 - \ln 4} = \dfrac{\ln 3}{\ln 4 - 2\ln 3}$

$= -1.354756$

14. (14pts) Pablo drives to a job interview in an hour. Returning along the same route, he feels more relaxed and drives 14mph slower, so it takes him an hour and a quarter.
a) How fast is Pablo driving on the way to and from the job interview?
b) How far did he travel one-way?

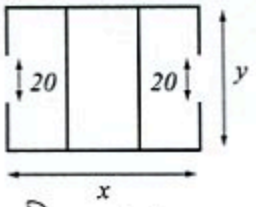
a) $d, r, 1\text{ hr}$ →
← $d, r-14, 1.25\text{ hrs}$

Drove to job interview: at 70 mph
From job interview: at 56 mph

b) $d = 70 \cdot 1 = 70$ miles.

Same, so
$$\begin{cases} d = r \cdot 1 \\ d = (r-14) \cdot 1.25 \end{cases}$$
$$r = (r-14) \cdot 1.25$$
$$r = 1.25r - 14 \cdot 1.25 \quad | -r + 17.5$$
$$17.5 = 0.25r$$
$$r = \frac{17.5}{0.25} = 70$$

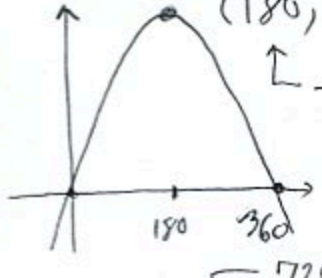
15. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1400 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



Domain:
Must have:
$y \geq 20$
$x \geq 0$
$720 - 2y \geq 0$
$2y \leq 720$
$y \leq 360$
Domain: $[20, 360]$

$$A = x \cdot y = (720-2y)y = \boxed{-2y^2 + 720y} = A(y)$$

$1400 = 2x + 2y + 2(y-20)$
$1400 = 2x + 4y - 40$
$2x = 1440 - 4y \quad | \div 2$
$x = 720 - 2y$



(180, 64,800)
$-\frac{b}{2a} = -\frac{720}{2(-2)} = 180$
$720 - 2 \cdot 180$

Dimensions: 180×360 ft
Max area: 64,800 sq. ft.

16. (12pts) The population of Splodaton was 24,000 in 2011 and 30,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2011. Graph it on paper.
b) Find the predicted population in the year 2022.

time	Pop.
2011 ($t=0$)	24 000
2015 ($t=4$)	30 000

a) $P(t) = 24e^{kt}$ (in thousands)

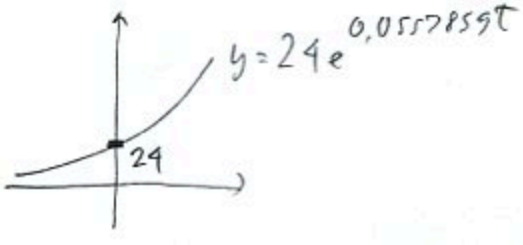
$P(4) = 30$

$24e^{k\cdot 4} = 30$

$e^{4k} = \frac{30}{24} = 1.25$ | ln

$4k = \ln 1.25$

$k = \frac{\ln 1.25}{4} = 0.0557859$



b) Pop. in 2022 is $P(11)$

$P(11) = 24 \cdot e^{0.0557859 \cdot 11}$

$= 44.331638$

About 44,332 people in 2022

Bonus (10pts) Betty has a total of \$7000 invested in two accounts, one bearing 5% and the other 8% interest. She notices that if she reversed the amounts invested in each account, she would have \$78 more in interest over a year. How much is invested in each account?

Let x - amount invested at 5%

$7000 - x$ = amount invested at 8%

Current interest is: $0.05 \cdot x + 0.08(7000 - x)$

Interest if amounts are swapped: $0.08x + 0.05(7000 - x)$ is 78 more than

$0.08x + 0.05(7000 - x) = 0.05x + 0.08(7000 - x) + 78$

$0.08x + 350 - 0.05x = 0.05x + 560 - 0.08x + 78$

$0.03x + 350 = -0.03x + 638$ | $+0.03x - 350$

$0.06x = 288$

$x = \frac{288}{0.06} = 4800$

\$4800 in acct at 5% interest

\$2200 ____ 8% ____