**College Algebra — Exam 4**
**MAT 140, Fall 2017 — D. Ivanšić**

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*Show all your work!*

**1.** (8pts) Evaluate without using the calculator:

$\log_4 256 = 4$ $\quad 4^? = 256$

$\log_3 \frac{1}{9} = -2$ $\quad 3^? = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$\log_a \sqrt[3]{a^5} = \frac{5}{3}$ $\quad a^? = a^{\frac{5}{3}}$

$\log_{b^2} b^{10} = 5$ $\quad (b^2)^? = b^{10}$

**2.** (4pts) Use the change-of-base formula and your calculator to find $\log_3 10$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_3 10 = \frac{\log 10}{\log 3} = \frac{1}{\log 3} = 2.095903$

**3.** (5pts) If $\log_a 3 = u$ and $\log_a 7 = v$, express in terms of $u$ and $v$:

$\log_a 49 = \log_a 7^2$
$= 2\log_a 7$
$= 2v$

$\log_a \frac{\sqrt{3}}{7} = \log_a \sqrt{3} - \log_a 7$
$= \frac{1}{2}\log_a 3 - \log_a 7$
$= \frac{u}{2} - v$

**4.** (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_5 \frac{125y^3}{\sqrt[5]{x^6}} = \log_5 125 + \log_5 y^3 - \log_5 \sqrt[5]{x^6}$
$= 3 + 3\log_5 y - \frac{6}{5}\log_5 x$

**5.** (6pts) Write as a single logarithm. Simplify if possible.

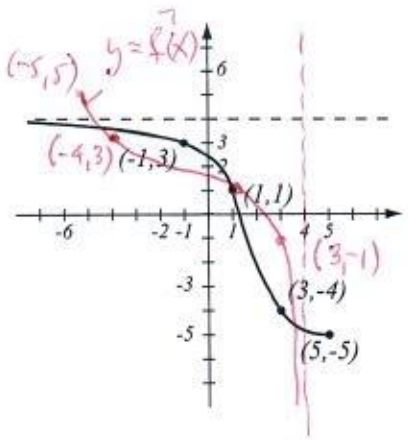
$2\log_7(x^{-4}y^4) - 3\log_7(x^2y^{-3}) = \log_7 (x^{-4}y^4)^2 - \log_7 (x^2y^{-3})^3$
$= \log_7 \frac{x^{-8}y^8}{x^6 y^{-9}} = \log_7 (x^{-14}y^{17}) = \log_7 \frac{y^{17}}{x^{14}}$

**6.** (4pts) Simplify.

$\log_8 8^{\sqrt{5}} = \sqrt{5}$

$e^{\ln(3-x^2)} = 3 - x^2$

**7.** (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points, and showing any asymptotes.




a) Yes - it passes the horizontal line test

**8.** (8pts) Let $f(x) = 4 + \sqrt{x+3}$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f^{-1}$.

a) $y = 4 + \sqrt{x+3}$

$y - 4 = \sqrt{x+3}$ |$^2$

$(y-4)^2 = x+3$

$x = (y-4)^2 - 3$

$f^{-1}(y) = (y-4)^2 - 3$

$= y^2 - 8y + 16 - 3$

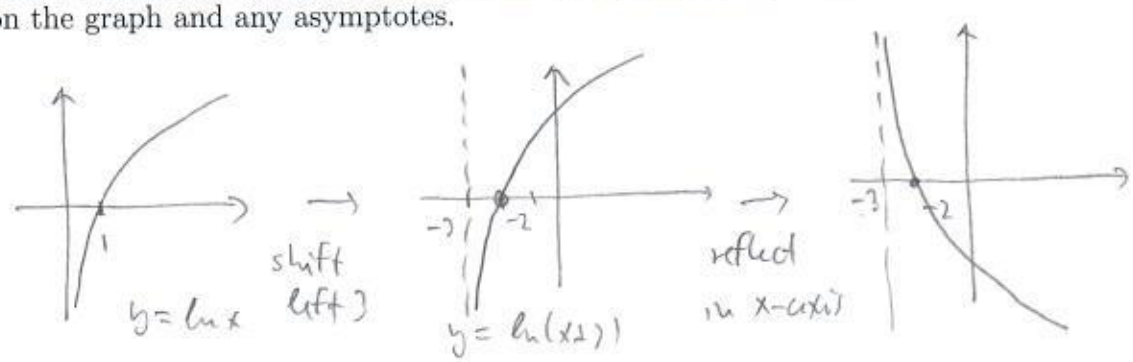
$= y^2 - 8y + 13$

b) range $f^{-1}$ = domain $f$ = $[-3, \infty)$

must have $x+3 \geq 0$

$x \geq -3$

**9.** (6pts) Using transformations, draw the graph of $f(x) = -\ln(x+3)$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.



**10.** (3pts) Find the domain of the function $f(x) = \log_{11}(3 - 5x)$ and write it in interval notation.

Must have $3 - 5x > 0$

$3 > 5x$

$x < \frac{3}{5}$

$(-\infty, \frac{3}{5})$

**11.** (8pts) How much should you invest in an account bearing 2.4%, compounded monthly, if you wish to have \$2,000 in four years?

$A = P(1 + \frac{r}{n})^{nt}$

$2000 = P(1 + \frac{0.024}{12})^{12 \cdot 4}$

$2000 = P(1.002)^{48}$

$2000 = P \cdot 1.100654$

$P = \frac{2000}{1.100654} = 1817.10$

Solve the equations.

**12.** (6pts) $25^{x+1} = \left(\frac{1}{5}\right)^{2x+4}$

$(5^2)^{x+1} = (5^{-1})^{2x+4}$

$5^{2x+2} = 5^{-2x-4}$

$2x + 2 = -2x - 4$

$4x = -6$

$x = -\frac{3}{2}$

**13.** (8pts) $3^{2x+1} = 4^{9-x}$ | ln

$\ln 3^{2x+1} = \ln 4^{9-x}$

$(2x+1)\ln 3 = (9-x)\ln 4$

$2x \ln 3 + \ln 3 = 9\ln 4 - x \ln 4$

$2x \ln 3 + x \ln 4 = 9\ln 4 - \ln 3$

$x(2\ln 3 + \ln 4) = 9\ln 4 - \ln 3$

$x = \frac{9\ln 4 - \ln 3}{2\ln 3 + \ln 4} = 3.175102$

**14.** (10pts) $\log_3(x - 8) + \log_3(2x + 5) = 4$

$\log_3((x-8)(2x+5)) = 4$ | $3^{-}$

$3^{\log_3 (x-8)(2x+5)} = 3^4$

$(x-8)(2x+5) = 81$

$2x^2 - 11x - 40 = 81$

$2x^2 - 11x - 121 = 0$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \cdot 2 \cdot (121)}}{2 \cdot 2} = \frac{11 \pm \sqrt{1089}}{4}$

$= \frac{11 \pm 33}{4} = 11, -\frac{22}{4} = 11, -\frac{11}{2}$

$x = 11$

↑ gives neg. no. in $\log_3(-\frac{11}{2} - 8)$, so not a solution

**15.** (12pts) The population of Splodaton was 35,000 in 2009 and 42,000 in 2014. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find $k$ and write the function that describes the population at time $t$ years since 2009. Graph it on paper.
b) Find the predicted population in the year 2019.

a) $P(t) = 35e^{kt}$

| | P | t |
|---|---|---|
| 2009 | 35 | 0 |
| 2014 | 42 | 5 |

$P(5) = 42$

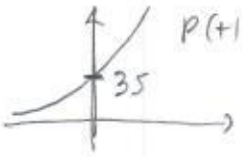
$42 = 35e^{k\cdot 5} \mid \div 35$

$\frac{42}{35} = e^{5k} \mid \ln$

$\ln 1.2 = 5k$

$k = \frac{\ln 1.2}{5} = 0.0364643$

$P(t) = 35e^{0.0364643t}$




b) 2019 corresponds to $t = 10$

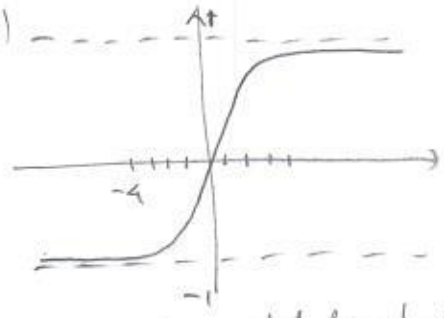
$P(10) = 35e^{0.0364643 \cdot 10}$

$= 50.4$

It will have 50,400 people in 2019

**Bonus** (10pts) Let $f(x) = \dfrac{e^x - 1}{e^x + 1}$.
a) Graph the function (sketch on paper!). Explain why it is one-to-one.
b) Find the formula for $f^{-1}(x)$.

a)




Passes horizontal line test, so it is one-to-one

b) $y = \dfrac{e^x - 1}{e^x + 1}$ solve for $x$

$y(e^x + 1) = e^x - 1$

$ye^x + y = e^x - 1$

$ye^x - e^x = -y - 1$

$e^x(y - 1) = -y - 1$

$e^x = \dfrac{-y-1}{y-1} = \dfrac{y+1}{1-y} \mid \ln$

$x = \ln\left(\dfrac{y+1}{1-y}\right)$