College Algebra — Exam 3
MAT 140, Fall 2017 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(2-3i)^2 - 3 - 5i = 4-12i+\underbrace{9i^2}_{=-9}-3-5i = -8-17i$

2. (5pts) $\dfrac{3+7i}{1+2i} = \dfrac{3+7i}{1+2i}\cdot\dfrac{1-2i}{1-2i} = \dfrac{3-6i+7i-\overbrace{14i^2}^{-14}}{1^2-\underbrace{4i^2}_{-4}} = \dfrac{17+i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{175} = i^{172}\cdot i^3 = (\underbrace{i^4}_{=1})^{43}\cdot i^3 = 1\cdot i\cdot i\cdot i = -i$

4. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 20 = 0 \quad | +4^2$

$x^2 - 2\cdot x\cdot 4 + 4^2 + 20 = 4^2$

$(x-4)^2 = 16-20$

$(x-4)^2 = -4$

$x-4 = \pm 2i$

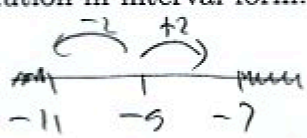
$x = 4 \pm 2i$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x+9| \geq 2$

$|x-(-9)| \geq 2$

distance from x to $-9 \geq 2$



$(-\infty, -11] \cup [-7, \infty)$

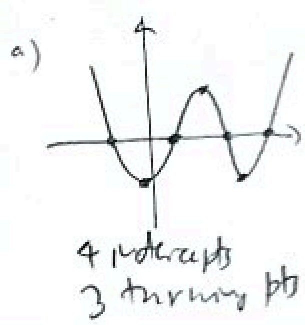
6. (6pts) Let $P(x)$ be a polynomial of degree 4. like W or M

a) Draw a graph of P that has the maximal number of x-intercepts and turning points.

b) Draw a graph of P that has exactly 3 x-intercepts.

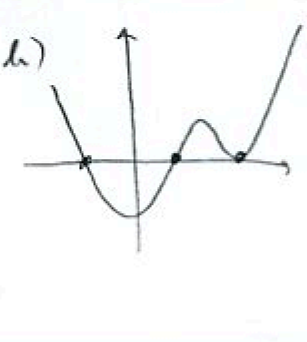
c) Can the graph of P have no turning points? Justify.

a)



4 intercepts
3 turning pts

b)



c) No - since general shape is W or M it has to have at least one min or max.

7. (12pts) The quadratic function $f(x) = x^2 - 6x - 16$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

y-int: $f(0) = -16$

x-int: $x^2 - 6x - 16 = 0$

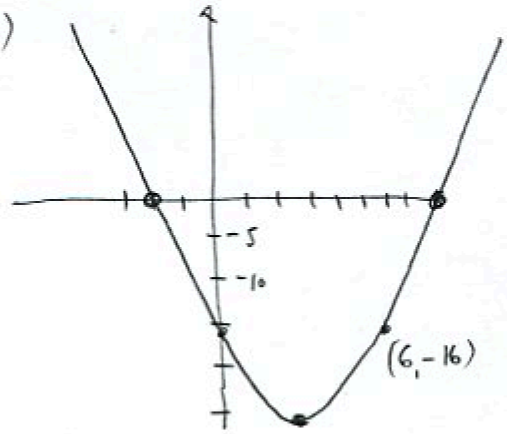
$(x-8)(x+2) = 0$

$x = 8, -2$

b) vertex: $h = -\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$

$k = f(3) = 3^2 - 6 \cdot 3 - 16$
$= 9 - 18 - 16$
$= -25$

c)



Solve the equations:

8. (8pts) $\frac{x+4}{x-6} + \frac{x^2 - 15x - 36}{x^2 - 3x - 18} = \frac{x+1}{x+3}$ $\Big| \cdot (x-6)(x+3)$

$(x-6)(x+3)$

$\frac{x+4}{\cancel{x-6}}\cancel{(x-6)}(x+3) + \frac{x^2 - 15x - 36}{\cancel{(x-6)}\cancel{(x+3)}}\cancel{(x-6)}\cancel{(x+3)} = \frac{x+1}{\cancel{x+3}}(x-6)\cancel{(x+3)}$

$x^2 + 7x + 12 + \cancel{x^2} - 15x - 36 = \cancel{x^2} - 5x - 6$

$x^2 - 8x - 24 = -5x - 6 \quad | + 5x + 6$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = 6, -3$

Both give 0 in denominator, so $\boxed{\text{no solution}}$.

9. (8pts) $2\sqrt{x+11} - 3 = x$

$2\sqrt{x+11} = x+3 \quad)^2$

$4(x+11) = x^2 + 6x + 9 \quad | -4x - 44$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x = \cancel{-7}, 5$

not a sol.

Check: $2\sqrt{-7+11} - 3 \stackrel{?}{=} -7$

$4 - 3 = -7$ no

$2\sqrt{5+11} - 3 \stackrel{?}{=} 5$

$8 - 3 \stackrel{?}{=} 5$ yes

$x = 5$ only solution

10. (14pts) The polynomial $f(x) = \frac{1}{10}(x+5)^2(x-7)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

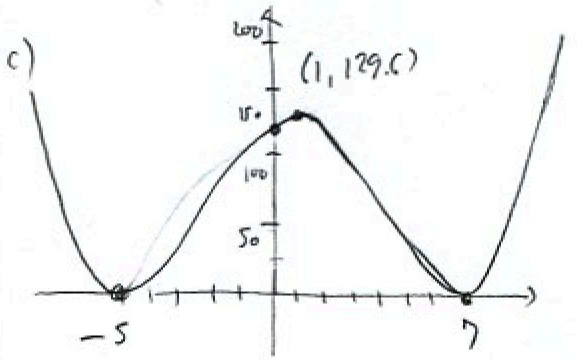
a) like $\frac{1}{10}(x..)^2(x..)^2 = \frac{1}{10}x^4$ U

b)

zero	-5	7
mult.	2	2

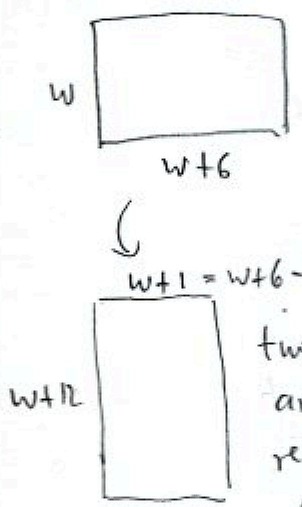
y-int:
$f(0) = \frac{1}{10} \cdot 25 \cdot 49 = 122.5$

c)



d) Turning points:
$(-5, 0)$, $(7, 0)$
$(1, 129.6)$

11. (12pts) In a rectangle, length is 6 inches more than the width. If we increase the width by 12 inches and decrease the length by 5 inches we arrive at a rectangle whose area is twice the area of the original rectangle. What are the dimensions of the original rectangle?



twice the area of rectangle above

2·area of $w \times (w+6)$ = area of $(w+12) \times (w+1)$ rectangle

$2w(w+6) = (w+12)(w+1)$

$2w^2 + 12w = w^2 + 13w + 12$

$w^2 - w - 12 = 0$

$(w-4)(w+3) = 0$

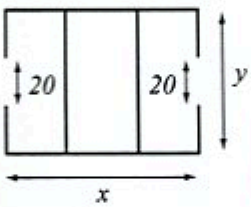
$w = -3, 4$

can't have $w < 0$

$w = 4$ only solution

4×10 rectangle

12. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1000 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



a) $A = xy = x\left(260 - \frac{1}{2}x\right) = -\frac{1}{2}x^2 + 260x$

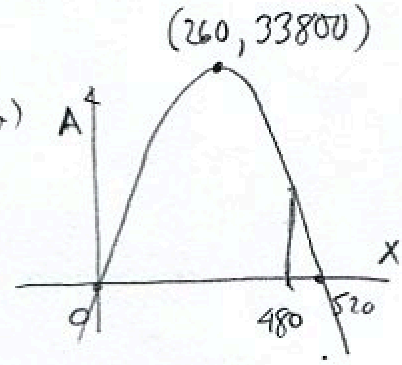
$2x + 2y + 2(y-20) = 1000$

$2x + 4y - 40 = 1000$

$4y = 1040 - 2x$

$y = 260 - \frac{1}{2}x$

b)



domain: Must have:

$x \geq 0$

$y \geq 20$

$260 - \frac{1}{2}x \geq 20$

$240 \geq \frac{1}{2}x$

$480 \geq x$

$[0, 480]$

$h = -\frac{b}{2a} = -\frac{260}{2\cdot(-\frac{1}{2})} = 260$

$k = -\frac{1}{2}260^2 + 260\cdot 260 = 33{,}800$

Dimensions: 260×130 ft

Max area: $33{,}800$ ft^2

Bonus. (10pts) Find $\sqrt{i}$, that is, find all complex numbers $x + yi$ so that $(x + yi)^2 = i$. To solve this equation, expand the left side, and solve for x and y using the fact that real and imaginary parts of both sides must be equal. Keep in mind that both x and y are real numbers.

$(x+yi)^2 = i$

$x^2 + 2xyi + y^2i^2 = i$

$(x^2 - y^2) + 2xyi = 0 + 1i$

Thus $x^2 - y^2 = 0$

$2xy = 1$

$x^2 - y^2 = 0$

$x^2 = y^2$

$y = \pm x$

put in $2xy = 1$

If $y = x$:

$2x \cdot x = 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

If $y = -x$

$2x\cdot(-x) = 1$

$x^2 = -\frac{1}{2} < 0$

has no real solution

Get $\left.\begin{matrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \end{matrix}\right\}$ are both solutions