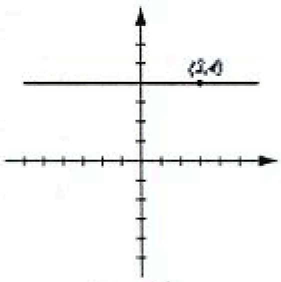
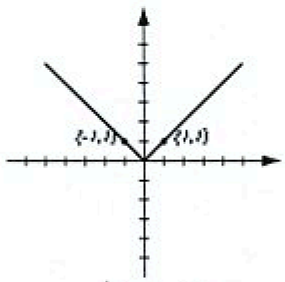
College Algebra — Exam 2 **MAT 140, Fall 2017 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

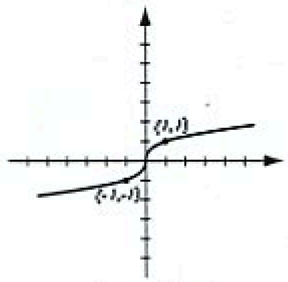
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



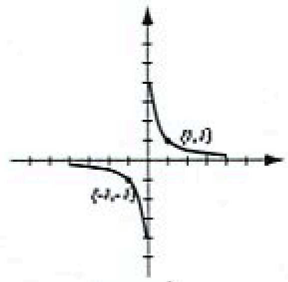
$y=4$



$y=|x|$



$y=\sqrt[3]{x}$



$y=\frac{1}{x}$

2. (21pts) Let $f(x)=2x+1$, $g(x)=\dfrac{x-5}{x^2-4}$.

Find the following (simplify where possible):

$(f+g)(1)= f(1)+g(1)$
$= 3+\frac{-4}{-3} = 3+\frac{4}{3} = \frac{13}{3}$

$(fg)(3) = f(3)\cdot g(3) = 7\cdot\frac{-2}{5} = -\frac{14}{5}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x+1}{\frac{x-5}{x^2-4}} = 2x+1\cdot\frac{x^2-4}{x-5}$
$= \frac{(2x+1)(x^2-4)}{x-5}$

$(f\circ g)(0) = f(g(0)) = f\left(\frac{-5}{-4}\right) = f\left(\frac{5}{4}\right)$
$= 2\cdot\frac{5}{4}+1 = \frac{5}{2}+1 = \frac{7}{2}$

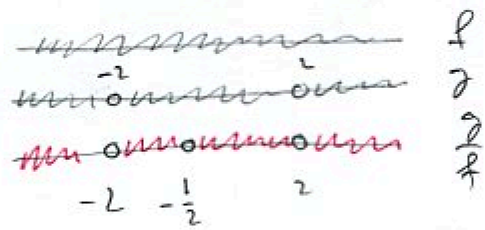
$(g\circ f)(x) = g(f(x)) = g(2x+1) = \frac{2x+1-5}{(2x+1)^2-4} = \frac{2x-4}{4x^2+4x+1-4} = \frac{2x-4}{4x^2+4x-3}$

The domain of $\frac{g}{f}$ in interval notation

domain of f: all reals

domain of g: can't have
$x^2-4=0$
$x^2=4$
$x=\pm 2$

can't have $f(x)=0$
$2x+1=0$
$x=-\frac{1}{2}$



$(-\infty,-2)\cup(-2,-\frac{1}{2})\cup(-\frac{1}{2},2)\cup(2,\infty)$

3. (6pts) Consider the function $h(x) = \dfrac{7}{x^2+3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

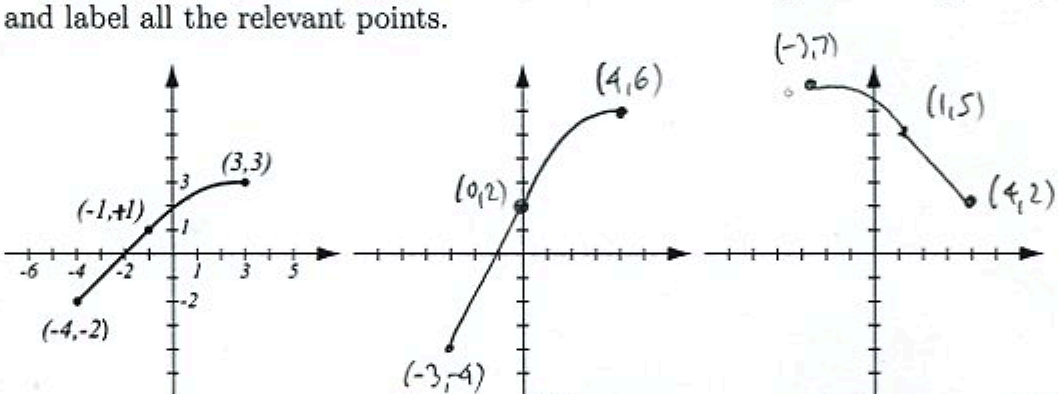
$g(x) = x^2+3$ $\qquad\qquad$ $g(x) = x^2$

$f(x) = \dfrac{7}{x}$ $\qquad\qquad$ $f(x) = \dfrac{7}{x+3}$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^2$, shifted down 2 units
b) shape of $y = \sqrt[3]{x}$, stretched horizontally by factor $\frac{1}{4}$, then reflected over the x-axis.

a) $y = x^2 - 2$ $\qquad\qquad$ b.) $y = -\sqrt[3]{\frac{1}{\frac{1}{4}}x} = -\sqrt[3]{4x}$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-1)$ and $f(-x)+4$ and label all the relevant points.



shift right 1
stretch vert., factor = 2

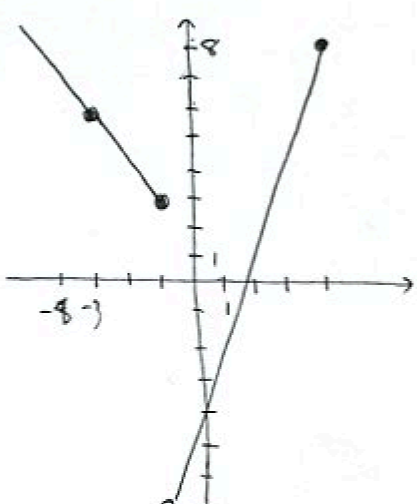
reflect in y-axis
shift up 4

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x, & \text{if } x \le -1 \\ 3x-4, & \text{if } -1 < x \le 4 \end{cases}$$

x	$-2x$
-1	2
-3	6

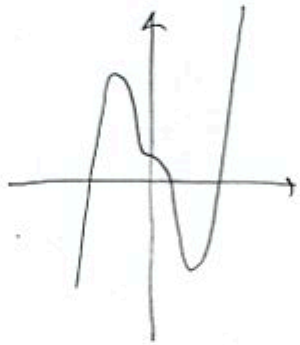
x	$3x-4$
-1	-7
4	8



7. (7pts) For the function $f(x) = x^5 - 7x^3 + 4$:
a) Determine algebraically whether it is odd, even, or neither.
b) Use the calculator to draw its graph here and verify your conclusion by stating symmetry.

$$f(-x) = (-x)^5 - 7(-x)^3 + 4$$
$$= -x^5 - 7(-x^3) + 4$$
$$= -x^5 + 7x^3 + 4 \neq f(x)$$
$$\neq -f(x)$$

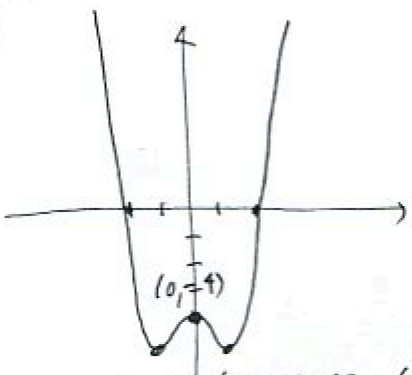
neither



no symmetry wrt origin or y-axis

8. (20pts) Let $f(x) = x^4 - 3x^2 - 4$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

a)



(−1.224747, −6.25) (1.224747, −6.25)

b) $f(-x) = (-x)^4 - 3(-x)^2 - 4$
$= x^4 - 3x^2 - 4 = f(x)$

so even

c) symmetric wrt y-axis

d) $f(-1.224747) = -6.25$
$f(1.224747) = -6.25$ are local minima

$f(0) = -4$ is a local maximum

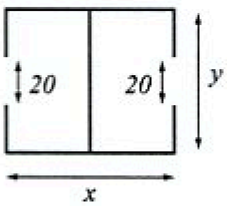
e) Increasing on

$(-1.224747, 0)$ and $(1.224747, \infty)$

Decreasing on

$(-\infty, -1.224747)$ and $(0, 1.224747)$

9. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. The warehouse is to have area 40,000 square feet and the company's goal is to minimize the total length of the walls.
a) Express the total wall length as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the warehouse that has the smallest total wall length and what is the minimal wall length?



a) $xy = 40000 = x$

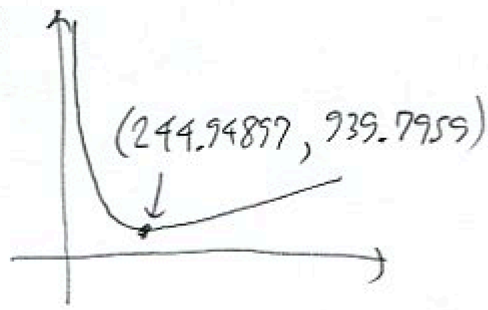
$y = \frac{40000}{x}$

$l = 2x + 2(y-20) + y$

$= 2x + 3y - 40$

$l(x) = 2x + \frac{120000}{x} - 40$

b)



Domain: $x > 0$

$y \geq 20$

$\frac{40000}{x} \geq 20$

$\frac{40000}{20} \geq x$

$x \leq 2000$

$(0, 2000]$

Dimensions are

$244.94897 \times 163.299316$

Smallest length is 939.7959 ft

Bonus. (10pts) Let $f(x) = x^2 + 2x - 4$ and $g(x) = \sqrt{x+5} - 1$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+5} - 1) = (\sqrt{x+5} - 1)^2 + 2(\sqrt{x+5} - 1) - 4$

$= \sqrt{x+5}^2 - 2 \cdot \cancel{\sqrt{x+5}} \cdot 1 + 1 + \cancel{2\sqrt{x+5}} - 2 - 4$

$= x + 5 - 5 = x$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 2x - 4) = \sqrt{x^2 + 2x - 4 + 5} - 1$

$= \sqrt{x^2 + 2x + 1} - 1 = \sqrt{(x+1)^2} - 1 = x + 1 - 1 = x$