Lebesgue Integrals

Type A problems (5pts each)

For the following functions and domains, determine if f is integrable over E. If it is, find the integral. Justify your reasoning with theorems, but if you do more than one of A1–A5, you may not appeal to the same theorem more than once.

A1.
$$f(x) = \frac{1}{x^p}, p > 1, E = [1, \infty)$$

A2. $f(x) = \frac{1}{x^p}, 0
A3. $f(x) = \frac{1}{x^p}, p > 1, E = (0, 1]$
A4. $f(x) = \frac{1}{x^p}, 0
A5. $f(x) = e^x, E = (-\infty, 0]$
A6. Show that $f(x) = \frac{1}{x^2} \sin x$ is integrable over $[1, \infty)$.$$

TYPE B PROBLEMS (8PTS EACH)

B1. Show that $f(x) = \frac{1}{x}\sin(\pi x)$ is not integrable over $[1,\infty)$, but that $\lim_{n\to\infty}\int_{[1,n]} f$ exists.