College Algebra — Joysheet 11
MAT 140, Spring 2016 — D. Ivanšić

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Covers: 5.4–5.6 *Show all your work!*

1. (5pts) If $\log_a 5 = 0.594316$ and $\log_a 8 = 0.767874$, find (show how you obtained your numbers):

$\log_a \frac{5}{8} = \log_a 5 - \log_a 8$
$= 0.594316 - 0.767874$
$= -0.173558$

$8 \cdot 25 = 8 \cdot 5^2$
$\log_a 200 = \log_a 8 + 2\log_a 5$
$= 0.767874 + 2 \cdot 0.594316$
$= 1.956506$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4(16u^3v^8) = \log_4 16 + \log_4 u^3 + \log_4 v^8$
$= 2 + 3\log_4 u + 8\log_4 v$

$\log_3 \sqrt[5]{\frac{81x^{-2}y^4}{x^{-5}y}} = \frac{1}{5}\log_3 \frac{81x^{-2}y^4}{x^{-5}y} = \frac{1}{5}\log_3(81x^3y^3) = \frac{1}{5}(\log_3 81 + \log_3 x^3 + \log_3 y^3)$
$= \frac{1}{5}(4 + 3\log_3 x + 3\log_3 y)$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\log(8x^2) - 4\log(3y^{\frac{3}{2}}) - \log(x^{\frac{5}{3}}) = \log(8x^2)^{1/3} - \log(3y^{\frac{3}{2}})^4 - \log x^{\frac{5}{3}}$
$= \log(2x^{\frac{2}{3}}) - \log(81y^6) - \log x^{\frac{5}{3}} = \log \frac{2x^{\frac{2}{3}}}{81y^6 \cdot x^{5/3}}$
$= \log \frac{2x^{-1}}{81y^6} = \log \frac{2}{81y^6 x}$

$3\ln(x-2) - 2\ln(x+5) + 4\ln(x^2+3x-10) =$
$= \ln(x-2)^3 - \ln(x+5)^2 + \ln(x^2+3x-10)^4$ $\quad (x+5)(x-2)$
$= \ln \frac{(x-2)^3 (x+5)^4 (x-2)^4}{(x+5)^2} = \ln((x-2)^7(x+5)^2)$

Solve the equations.

4. (5pts) $32^{2x+1} = \left(\frac{1}{8}\right)^{2x-5}$

$(2^5)^{2x+1} = (2^{-3})^{2x-5}$

$2^{10x+5} = 2^{-6x+15}$

$10x+5 = -6x+15$

$16x = 10$

$x = \frac{10}{16} = \frac{5}{8}$

5. (7pts) $7^{3-2x} = 3^{x+4}$ | ln

$\ln 7^{3-2x} = \ln 3^{x+4}$

$(3-2x)\ln 7 = (x+4)\ln 3$

$3\ln 7 - 2x\ln 7 = x\ln 3 + 4\ln 3$

$3\ln 7 - 4\ln 3 = x\ln 3 + 2x\ln 7$

$x(2\ln 7 + \ln 3) = 3\ln 7 - 4\ln 3$

$x = \frac{3\ln 7 - 4\ln 3}{2\ln 7 + \ln 3} = 0.28921$

6. (8pts) $2^x + 3 = 18 \cdot 2^{-x}$

Let $u = 2^x$

Then $2^{-x} = \frac{1}{u}$

$u + 3 = 18\cdot\frac{1}{u}$ | $\cdot u$

$u^2 + 3u = 18$

$u^2 + 3u - 18 = 0$

$(u+6)(u-3) = 0$

$u = -6$ or 3

$2^x = -6$

has no sol. ($2^x > 0$)

$2^x = 3$

$\ln 2^x = \ln 3$

$x\ln 2 = \ln 3$

$x = \frac{\ln 3}{\ln 2}$

7. (12pts) According to World Bank data, world population was 5.2827 billion in 1990 and 7.2607 billion in 2014. (Do you still feel unique?) Assume it grows exponentially.
a) Write the function describing the number $P(t)$ of people t years after 1990. Then find the exponential growth rate of world population.
b) Graph the function.
c) According to this model, when will the population reach 9 billion?

a) $P(t) = 5.2827\, e^{kt}$

$7.2607 = 5.2827\, e^{k\cdot 24}$

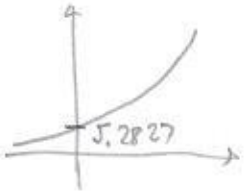
$1.374429\ldots = e^{k\cdot 24}$ | ln

$\ln(1.37\ldots) = k\cdot 24$

$k = \frac{\ln(1.37\ldots)}{24} = 0.0132516$

$P(t) = 5.2827\, e^{0.0132516\,t}$

b)



c) $9 = 5.2827\, e^{0.0132516t}$ | $\div 5.2827$

$1.70\ldots = e^{0.0132516t}$ | ln

$\ln(1.70\ldots) = 0.0132516t$

$t = \frac{\ln(1.70\ldots)}{0.0132516} = 40.205438$

In about 2030