College Algebra — Joysheet 8
MAT 140, Spring 2016 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $2 + 3i - 2i(4 - i) =$ $2+3i-8i+2i^2$

$= 2 - 5i - 2 = -5i$

2. (6pts) $\frac{1-2i}{4+9i} = \frac{1-2i}{4+9i} \cdot \frac{4-9i}{4-9i} = \frac{4-8i-9i+18i^2}{4^2-(9i)^2} = \frac{4-17i-18}{16-(-81)}$

$= \frac{-14-17i}{97} = -\frac{14}{97} - \frac{17}{97}i$

3. (4pts) Simplify and justify your answer.

$i^{135} = i^{134} \cdot i = \underbrace{(i^2)^{67}}_{-1} \cdot i = -i$

4. (6pts) Starting from rest, an object falling t seconds travels approximately $s = 5t^2$ meters. How long would it take for a TV set to fall from a tower 80 meters tall?

$80 = 5t^2$ Only $t=4$ fits the context

$t^2 = 16$

$t = \pm 4$

5. (8pts) Solve the equation: $6x^4 + 13x^2 - 5 = 0$

Let $u = x^2$ $6u^2 + 13u - 5 = 0$

$u = \frac{-13 \pm \sqrt{13^2 - 4 \cdot 6 \cdot (-5)}}{2 \cdot 6}$

$= \frac{-13 \pm \sqrt{169 + 120}}{12}$

$= \frac{-13 \pm \sqrt{289}}{12} = \frac{-13 \pm 17}{12} = -\frac{30}{12}, \frac{4}{12}$

$= -\frac{5}{2}, \frac{1}{3}$

$x^2 = -\frac{5}{2}$ $x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{5}{2}}\,i$ $x = \pm\sqrt{\frac{1}{3}}$

6. (6pts) Solve by completing the square.

$x^2 + 22x + 5 = 0$ $\quad | + 11^2$

$x^2 + 2 \cdot x \cdot 11 + 11^2 + 5 = 11^2$

$(x + 11)^2 + 5 = 121$

$(x + 11)^2 = 116$

$x + 11 = \pm\sqrt{116}$

± 4.29

$x = -11 \pm 2\sqrt{29}$

7. (12pts) The quadratic function $f(x) = 4x^2 - 4x - 15$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) $4x^2 - 4x - 15 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot (-15)}}{2 \cdot 4}$$

$$= \frac{4 \pm \sqrt{16 + 240}}{8} = \frac{4 \pm \sqrt{256}}{8}$$

$$= \frac{4 \pm 16}{8} = \frac{20}{8}, -\frac{12}{8} = \frac{5}{2}, -\frac{3}{2}$$

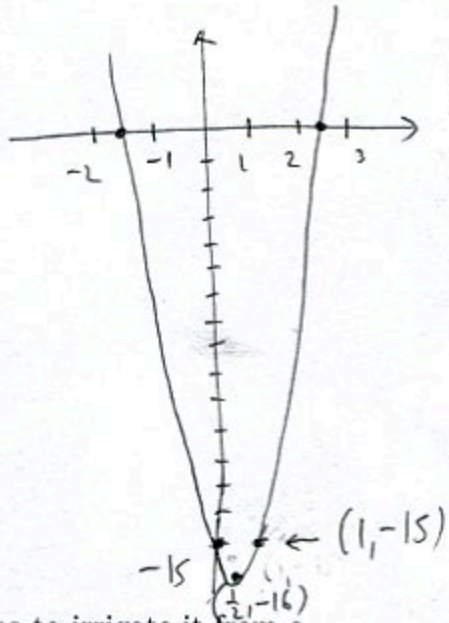
y-int: $f(0) = -15$

b) $h = -\frac{b}{2a} = -\frac{-4}{2 \cdot 4}$

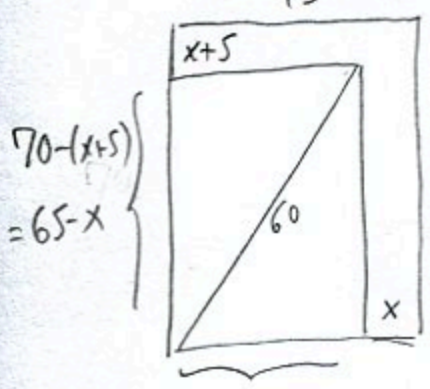
$= \frac{1}{2}$

$k = 4 \cdot \left(\frac{1}{2}\right)^2 - 4 \cdot \frac{1}{2} - 15$

$= 1 - 2 - 15 = -16$



8. (14pts) Cory has a plot of land that is 45 by 70 meters. He plans to irrigate it from a well inside the plot (whose location is not known in advance) with a hose 60 meters long. In order to be sure the hose can reach all parts of the plot he decides to reduce the dimensions of the plot: the 45-meter side by a certain amount, and the 70-meter side by 5 meters more than the amount the 45-meter side is reduced. By how much will Cory reduce the 45- and 70-meter sides to be sure that the hose can reach every part of the plot, wherever the well is in the plot? (Note that this is equivalent to having the diagonal of the new plot be 60 meters, since the diagonal represents the greatest distance between any two points in a rectangle.)



Pythagorean Theorem:

$(65-x)^2 + (45-x)^2 = 60^2$

6250
7600
2650

$4225 - 130x + x^2 + 2025 - 90x + x^2 = 3600 \quad | -3600$

$2x^2 - 220x + 2650 = 0 \quad | \div 2$

$x^2 - 110x + 1325 = 0$

$$x = \frac{-(-110) \pm \sqrt{(-110)^2 - 4 \cdot 1 \cdot 1325}}{2 \cdot 1}$$

$$= \frac{110 \pm \sqrt{12100 - 5300}}{2} = \frac{110 \pm \sqrt{6800}}{2} = \frac{110 \pm 20\sqrt{17}}{2} = 55 \pm 10\sqrt{17}$$

4·17·100

too big

$= 96.231056$

13.768944

Reduce:
45m side by 13.768944
70m side by 18.768944