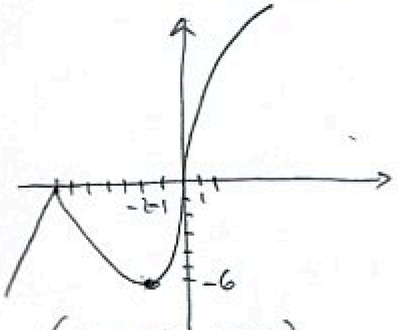
College Algebra — Joysheet 6 **MAT 140, Spring 2016 — D. Ivanšić**	**Name:** Saul Ocean *Covers: 2.1, 2.2, 2.3 Show all your work!*

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = |x+7| \cdot \sqrt[3]{x}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



$(-1.75, -6.326623)$

a) Local max is $f(7) = 0$

Local min is $f(-1.75) = -6.326623$

b) Incr. on $(-\infty, -7)$ and $(1.75, \infty)$

Decr. on $(-7, -1.75)$

2. (20pts) Let $f(x) = \sqrt{2x+7}$, $g(x) = \dfrac{2x}{x^2+5x-14}$. Find the following (simplify where possible):

$(f-g)(1) = f(1) - g(1) = \sqrt{9} - \dfrac{2}{1+5-14}$

$= 3 + \dfrac{1}{4} = \dfrac{13}{4}$

$(fg)(-2) = f(-2)\, g(-2) = \sqrt{3} \cdot \dfrac{2 \cdot (-2)}{(-2)^2 - 10 - 14}$

$= \dfrac{-4\sqrt{3}}{-20} = \dfrac{\sqrt{3}}{5}$

$\dfrac{g}{f}(x) = \dfrac{g(x)}{f(x)} = \dfrac{\frac{2x}{x^2+5x-14}}{\sqrt{2x+7}}$

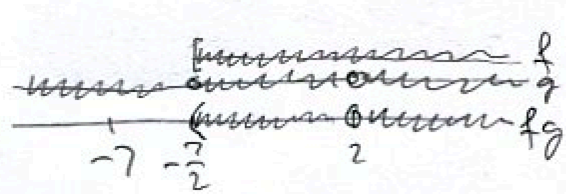
$= \dfrac{2x}{x^2+5x-14} \cdot \dfrac{1}{\sqrt{2x+7}} = \dfrac{2x}{(x^2+5x-14)\sqrt{2x+7}}$

$(f \circ g)(2) = f(g(2)) = f\left(\dfrac{4}{4+10-14}\right) = f\left(\dfrac{4}{0}\right)$

So not defind $\qquad$ not def.

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{2x+7}\right) = \dfrac{2\sqrt{2x+7}}{\underbrace{\sqrt{2x+7}^2}_{=2x+7} + 5\sqrt{2x+7} - 14} = \dfrac{2\sqrt{2x+7}}{2x-7+5\sqrt{2x+7}}$

The domain of $(fg)(x)$ in interval notation



Domain of $f(x)$: must have

$2x+7 \geq 0$

$2x \geq -7$

$x \geq -\frac{7}{2}$

Domain of g:

Can't have;

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x=-7, 2$

Domain of fg is

$[-\frac{7}{2}, 2) \cup (2, \infty)$

3. (8pts) Consider the function $h(x) = \sqrt{x^2+5}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$g(x) = x^2+5$ $\qquad$ $g(x) = x^2$ $\qquad$ $g(x) = x^2+2$

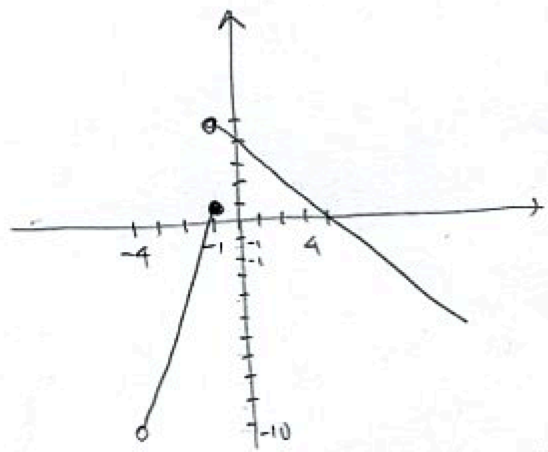
$f(x) = \sqrt{x}$ $\qquad$ $f(x) = \sqrt{x+5}$ $\qquad$ $f(x) = \sqrt{x+3}$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x+2, & \text{if } -4 < x \leq -1 \\ 4-x, & \text{if } -1 < x. \end{cases}$$

x	$3x+2$
-4	-10
-1	1

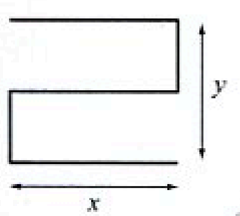
x	$4-x$
-1	5
4	0



5. (14pts) A self-storage business is adding a 1000-square-foot building with two large units that share a wall and have openings on opposite sides (see picture). They wish to minimize the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the building for which the total length of the walls is minimal?



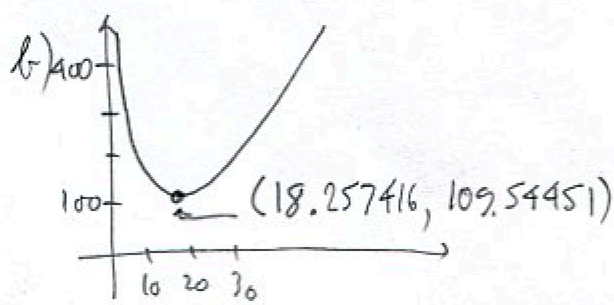
a) $L = 3x + y$

$xy = 1000$ (area condition)

So $y = \dfrac{1000}{x}$

So $L(x) = 3x + \dfrac{1000}{x}$

Domain: $x > 0$



The local min is $f(18.257416) = 109.54451$

Dimensions of the building are

$x \times y = 18.257416 \times 54.772265$

$\dfrac{1000}{18.257416}$