

Solve the equations.

1. (3pts) $3x - 7(2x + 1) = 5(x - 2) + 4$
 $3x - 14x - 7 = 5x - 10 + 4$
 $-11x - 7 = 5x - 6 \quad | +11x + 6$
 $-1 = 16x$
 $x = -\frac{1}{16}$

2. (6pts) $x^2 - 4 = 5x + 10$
 $x^2 - 5x - 14 = 0$
 $(x-7)(x+2) = 0$
 $x = -2, 7$

3. (5pts) Solve the equation for z:

$b^2z = az - 7 \quad | -az$
 $b^2z - az = -7$
 $z(b^2 - a) = -7$
 $z = -\frac{7}{b^2 - a} = \frac{7}{a - b^2}$

Simplify.

4. (8pts) $\frac{x-3}{4x^2+23x+15} - \frac{2x}{x^2-25} = \frac{x-3}{(4x+3)(x+5)} - \frac{2x}{(x-5)(x+5)}$
 $\frac{x-3}{4x^2+20x+3x+15} - \frac{2x}{x^2-25}$
 $\frac{x-3}{4x(x+5)+3(x+5)} - \frac{2x}{(x-5)(x+5)}$
 $\frac{x-3}{(4x+3)(x+5)} - \frac{2x}{(x-5)(x+5)}$
 $= \frac{(x-3)(x-5) - 2x(4x+3)}{(4x+3)(x+5)(x-5)}$
 $= \frac{x^2 - 8x + 15 - 8x^2 - 6x}{(4x+3)(x+5)(x-5)} = \frac{-7x^2 - 14x + 15}{(4x+3)(x+5)(x-5)}$
 prod = -105
 sum = -14
 no sol
 doesn't factor

5. (8pts) $\frac{\frac{3}{x} - \frac{x+1}{x-3}}{\frac{2x}{x-3} + \frac{x+4}{x}} = \frac{\frac{3(x-3) - (x+1)x}{x(x-3)}}{\frac{2x \cdot x + (x-3)(x+4)}{(x-3)x}} = \frac{3x-9-x^2-x}{x(x-3)} \cdot \frac{(x-3)x}{2x^2+x^2+x-12}$

$= \frac{-x^2 + 2x - 9}{3x^2 + x - 12}$
 don't factor
 prod = 9 } no sol,
 sum = 2 }
 prod = -36 } so they
 sum = 1 } don't factor

Simplify, showing intermediate steps. Assume variables can be any real numbers.

6. (2pts) $\sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$

7. (4pts) $\sqrt{45x^3y^4} = \sqrt{9 \cdot 5 \cdot x^2 \cdot x \cdot (y^2)^2} = 3\sqrt{5|x|y^4} = 3y^2|x|\sqrt{5x}$

8. (5pts) $\frac{\sqrt[3]{54x^7y^{10}}}{\sqrt[3]{16xy^2}} = \sqrt[3]{\frac{54x^7y^{10}}{16xy^2}} = \sqrt[3]{\frac{27x^6y^8}{8}} = \frac{\sqrt[3]{27(x^2)^3(y^2)^3y^2}}{\sqrt[3]{8}} = \frac{3x^2y^2\sqrt[3]{y^2}}{2}$

$y^2 \cdot y^2$ \uparrow since $y^2 \geq 0$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{(x^{\frac{2}{3}}y^{\frac{4}{3}})^{\frac{3}{2}}}{(x^{\frac{1}{3}}y^{-\frac{2}{3}})^2} = \frac{x^{\frac{2}{3} \cdot \frac{3}{2}} y^{\frac{4}{3} \cdot \frac{3}{2}}}{x^{\frac{1}{3} \cdot 2} y^{-\frac{2}{3} \cdot 2}} = \frac{x^{\frac{2}{3}} y^2}{x^{\frac{2}{3}} y^{-\frac{4}{3}}} = x^{\frac{2}{3} - \frac{2}{3}} y^{2 - (-\frac{4}{3})} = x^{\frac{1}{3}} y^{\frac{10}{3}} = \sqrt[3]{x} \sqrt[3]{y^{10}}$$

10. (6pts) Simplify.

$$\begin{aligned} (\sqrt{3} - 2)^2 - \sqrt{3}(2\sqrt{3} + 5) &= \sqrt{3}^2 - 2 \cdot \sqrt{3} \cdot 2 + 2^2 - 2\sqrt{3}^2 - 5\sqrt{3} \\ &= 3 - 4\sqrt{3} + 4 - 6 - 5\sqrt{3} \\ &= 1 - 9\sqrt{3} \end{aligned}$$

11. (5pts) Rationalize the denominator.

$$\begin{aligned} \frac{\sqrt{5} + 3\sqrt{2}}{\sqrt{5} - \sqrt{2}} \cdot \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}} &= \frac{\sqrt{5}^2 + 3\sqrt{2}\sqrt{5} + \sqrt{5}\sqrt{2} + 3\sqrt{2}^2}{\sqrt{5}^2 - \sqrt{2}^2} = \frac{5 + 3\sqrt{10} + \sqrt{10} + 6}{5 - 2} \\ &= \frac{11 + 4\sqrt{10}}{3} \end{aligned}$$