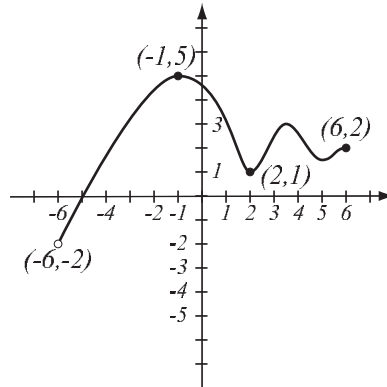**1.** (8pts)  Use the graph of the function $f$
at right to answer the following questions.

a) Find $f(-1)$ and $f(-6)$.

b) What is the domain of $f$?

c) What is the range of $f$?

d) What are the solutions
of the equation $f(x) = 4$?

**2.** (10pts)  Use your calculator
to accurately sketch the graph of
$y = x^3 - 6x^2 + 6x - 3$.  Draw
the graph here, and indicate units
on the axes. Find all the $x-$ and
$y-$intercepts (accuracy: 6 decimal
points).

**3.** (4pts)  Convert to scientific notation or a decimal number:

$4.171824 \times 10^6 =$                                    $0.0007459 =$

Use formulas to expand:

**4.** (3pts)  $(x - y^4)(x + y^4) =$

**5.** (4pts)  $(3s + 5t)^2 =$

**6.** (5pts)  Factor: $u^3 + 27 =$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

**7.** (2pts) $\sqrt{48} =$

**8.** (5pts) $\sqrt{125x^6y^3} =$

**9.** (8pts) Simplify.

$$\frac{x-1}{x^2-9} - \frac{4x}{x^2-4x-21} =$$

**10.** (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$$\frac{\left(x^9 y^{-\frac{3}{2}}\right)^{\frac{1}{3}}}{\left(x^{\frac{1}{2}} y^{\frac{3}{2}}\right)^5} =$$

**11.** (6pts) Rationalize the denominator.

$$\frac{2\sqrt{3}-5}{4+\sqrt{3}}$$

**12.** (5pts)  Solve the equation for $t$.

$ct - 5a = t + 1$

**13.** (8pts)  Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{x - 13}{x^2 + 6x - 40}$$

$$g(x) = \sqrt[3]{3x - 11}$$

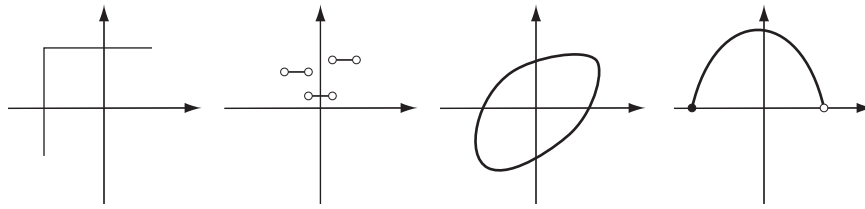**14.** (9pts)  Let $g(x) = 2x^2 + 3x - 7$. Find the following (simplify where appropriate).

$g(-2) =$

$g(-u) =$

$g(\sqrt{x + 5}) =$

$g(x + 5) =$

**15.** (5pts) Which of the folowing graphs are graphs of functions? Why?



**16.** (10pts) The diameter of a circle has endpoints $(-3, -2)$ and $(1, 4)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

**Bonus** (10pts) Find the coordinates $(x, y)$ of at least 4 points in the plane that lie on the curve with the equation $(x - 2)^2 + (y + 4)^2 = 10$. *(Hint: set one variable, and solve for the other; or draw the curve and infer some points from the picture.)*

**1.** (8pts)  The following are graphs of basic functions.  Write the equation of the graph under each one.
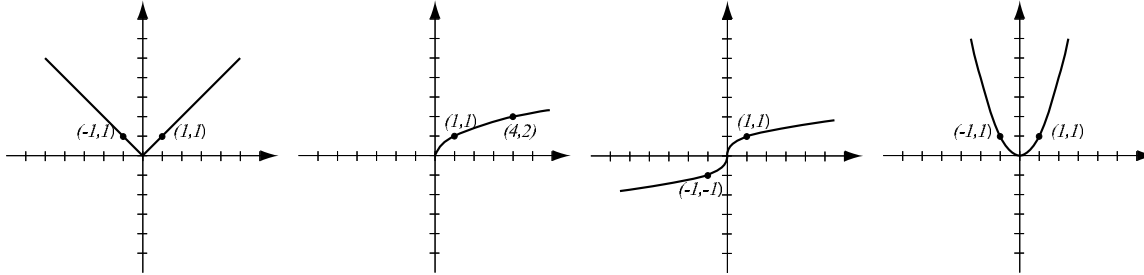


**2.** (14pts)  First find the equation of the line (in form $y = mx + b$) whose $x$-intercept is 3 and $y$-intercept is 2. Then find the equation of the line that is perpendicular to the first one and passes through the origin. Draw both lines.
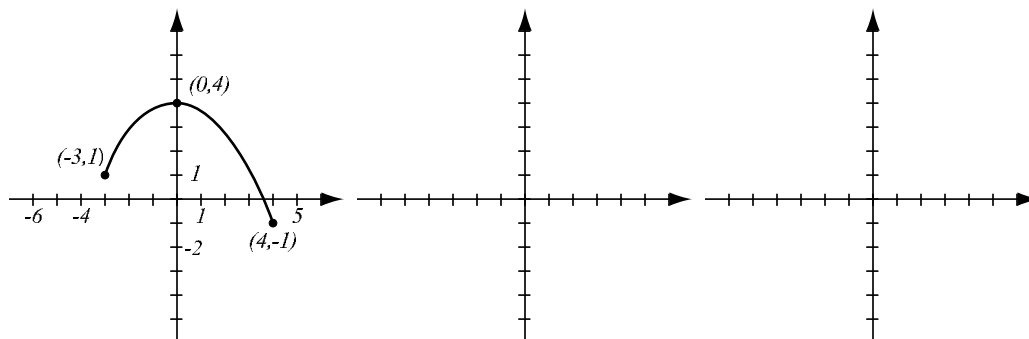
**3.** (5pts)  Solve the inequality and write your solution in interval notation.

$2x + 1 \leq 7$ or $3x - 6 > 8$

**4.** (8pts)  Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x + 1, & \text{if } -5 < x \leq -2 \\ -5 - 2x, & \text{if } -2 < x \leq 3 \end{cases}$$

**5.** (10pts)  The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)+1$ and $3f(x+1)$ and label all the relevant points.



**6.** (14pts)  Let $f(x) = x^2 + 1$, $g(x) = \dfrac{2}{\sqrt{3x+1}}$.

Find the following (simplify where possible):

$(f + g)(5) =$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $(fg)(x) =$

$(g \circ f)(-2) =$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $(f \circ g)(x) =$

The domain of $g$ in interval notation

**7.** (3pts)  Consider the function $h(x) = \dfrac{1}{x^3 - 17x}$. Find functions $f$ and $g$, neither of which is the "stupid" one, so that $h(x) = f(g(x))$.

**8.** (12pts)  Let $f(x) = x^3 - 3x^2 - 7x - 4$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of $f$ (on paper!).  Indicate scale on the graph.
b) Find the local maxima and minima for this function.
c ) State the intervals where the function is increasing and where it is decreasing.

**9.** (12pts)  Fiona is considering her options for a cab ride: the company Quicky Qabs charges a \$5 startup fee, which includes three miles of travel, and \$2 per mile after three miles, and the company Speedy Sedan charges a \$7 startup fee that includes five miles of travel, and \$2.50 per mile after five miles.  If Fiona will travel at least five miles, for which length of travel is Speedy Sedan better?

**10.** (14pts)  Greg drove one and a half hours from Paducah to Hopkinsville on a scenic route. On his way back, he took a route 13 miles shorter, so the trip lasted one hour and fifteen minutes. He drove at the same speed on both trips.
a) What was Greg's speed on both trips?
b) How long is the scenic route?

**Bonus.** (14pts) Eric wishes to enclose a rectangular pen for his dog along a wall of his house (there is no fence along the wall). The pen is to have area of 500 square feet and Eric wishes to minimize the length of fence used.
a) Express the length of the fence used as a function of the length of one of the sides $x$. What is the domain of this function?
b) Graph the length function in order to find the minimum. What are the dimensions of the pen that give the minimum fence length?
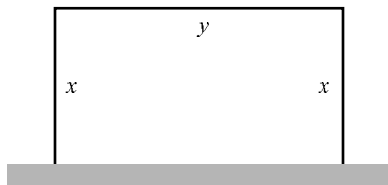
Simplify, so that the answer is in form $a + bi$.

**1.** (4pts)  $i(i + 1) - 2i(i - 1) =$

**2.** (6pts)  $\dfrac{4 - i}{2 + 5i} =$

**3.** (4pts)  Simplify and justify your answer.

$i^{114} =$

**4.** (6pts)  Solve the equation by completing the square.

$x^2 + 12x + 40 = 0$

**5.** (6pts)  Solve the inequality. Write the solution in interval form.

$|x + 9| \geq 2$

**6.** (6pts)  Let $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, and suppose $a > 0$ and $f < 0$.
a) State the maximum number of $x$-intercepts that the graph of $P$ may have.
b) State the maximum number of turning points that the graph of $P$ may have.
c) Draw a possible graph of $P$, if it has three $x$-intercepts and the maximal number of turning points.

**7.** (12pts) The quadratic function $f(x) = x^2 - 4x + 5$ is given. Do the following without using the calculator.

a) Find the $x$- and $y$-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

Solve the equations:

**8.** (8pts) $\dfrac{x^2 - x}{x^2 - 9} + \dfrac{4}{x + 3} = \dfrac{1}{x - 3}$

**9.** (8pts) $\sqrt{6x + 7} - x = 2$

**10.** (14pts) The polynomial $f(x) = \frac{1}{2}(x-4)^2(x+6)^2$ is given.
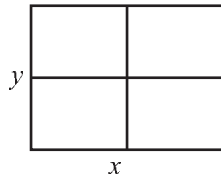a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the $y$-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of $f$ (yes, on paper!).
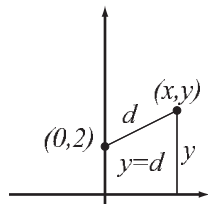d) Find all the turning points (i.e., local maxima and minima).

**11.** (12pts) In a rectangle, one side is 9cm longer than the other. If the shorter side is lengthened by 5cm, and the longer side shortened by 3cm, we get a rectangle with twice the area of the original one. What are the dimensions of the original rectangle?

**12.** (14pts)  A developer has budgeted enough money to build 1200 feet of walls in a small retail building consisting of two stores with back rooms, where the front of the store is not walled in (see picture).  The developer's goal is to maximize the total area of the building.
a) Express the area of the building as a function of the length of one of the sides $x$.  What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like).  What are the dimensions of the building that has the biggest possible area?



**Bonus.** (10pts) Recall that the distance between points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.  Consider the set of points $(x, y)$ that has equal distance to the point $(0, 2)$ and the $x$-axis (distance from a point to the $x$-axis is the $y$-coordinate of the point).  Write the equation that a point $(x, y)$ with this property has to satisfy, and simplify it to form $y = f(x)$.  You should get a parabola.  (This is the classical definition of a parabola: set of points whose distance to a fixed point and a fixed line is equal.)

**1.** (8pts)  Evaluate without using the calculator:

$\log_8 64 =$            $\log_3 \frac{1}{81} =$            $\log_a \sqrt[5]{a^2} =$            $\log_{b^4} \sqrt{b} =$

**2.** (4pts)  Use the change-of-base formula and your calculator to find $\log_5 6$ with accuracy 6 decimal places.  Show how you obtained your number.

**3.** (5pts)  If $\log_a 3 = c$ and $\log_a 7 = d$, express in terms of $c$ and $d$:

$\log_a 21 =$                                $\log_a \dfrac{27}{49} =$

**4.**  (6pts)   Write as a sum and/or difference of logarithms.  Express powers as factors. Simplify if possible.
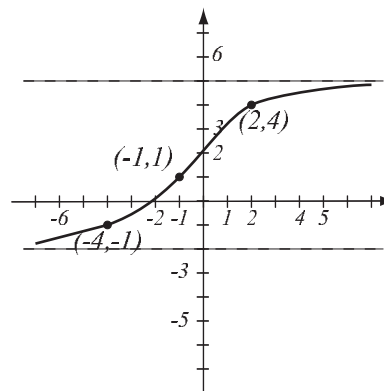
$\ln \dfrac{e^2 x^3}{\sqrt[3]{y^5}} =$

**5.** (12pts)  Write as a single logarithm. Simplify if possible.

$2\log_4(x^2 y^{-4}) + 3\log_4(x^{-2} y^3) =$

$3\log(x + 7) - 4\log(x^2 + 4x - 21) + 5\log(x - 3) =$

**6.** (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points, and showing any asymptotes.



**7.** (9pts) Let $f(x) = \dfrac{3-x}{x+2}$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f$.

**8.** (6pts) Using transformations, draw the graph of $f(x) = e^{x+3} - 4$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.

**9.** (6pts) Find the domain of the function $f(x) = \dfrac{\log_6(4x - 15)}{x - 4}$ and write it in interval notation.

**10.** (8pts) How much should you invest in an account bearing 3.66%, compounded monthly, if you wish to have \$3,000 in five years?

Solve the equations.

**11.** (8pts) $7^{2x-1} = 5^{x+2}$

**12.** (10pts) $3^{2x} - 6 \cdot 3^x = 18$

**13.** (12pts) The population of Spiriton was 95,000 in 2000 and 126,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.

a) Find $k$ and write the function that describes the population at time $t$ years since 2000. Graph it on paper.

b) Find the predicted population in the year 2021.

**Bonus** (10pts) What is better: depositing money into an account with interest rate 4.5%, compounded quarterly, or into an account with interest rate 4.4%, compounded monthly? (To determine this, calculate the amount at the end of 1 year, if $100 is deposited into either account.)

**1.** (8pts)  The following are graphs of basic functions.  Write the equation of the graph under each one.

*(1,1)*  *(-1,-1)*

*(-1,1)*  *(1,1)*

*(1,1)*  *(4,2)*

*(2,4)*  *(-1,1/2)*  *(1,2)*
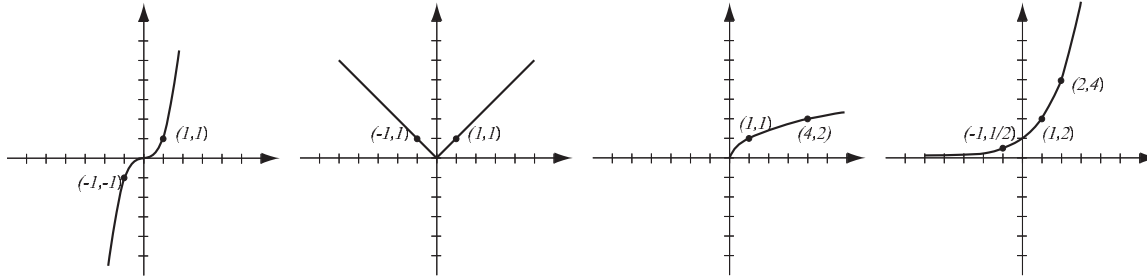
**2.** (8pts)  Use the graph of the function $f$ at right to answer the following questions.

a) What is the domain of $f$?

b) What is the range of $f$?

c) Find the intervals of increase of $f$.

d) What are the solutions
   of the equation $f(x) = -4$?

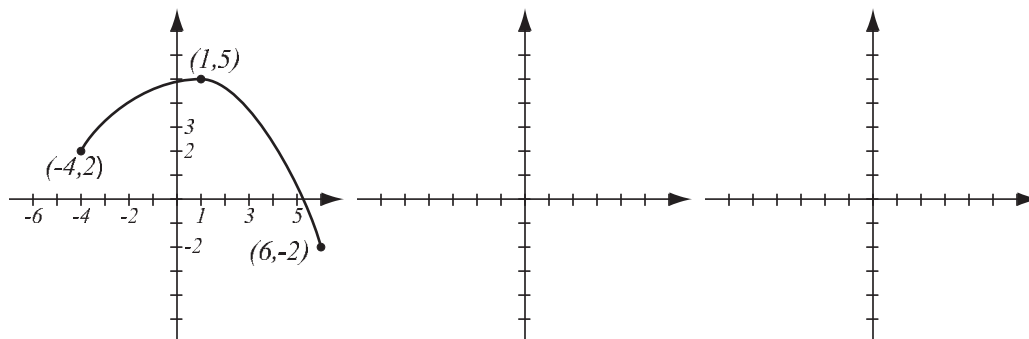*(-4,4)*  *(-6,1)*  *(5,-3)*  *(1,-5)*
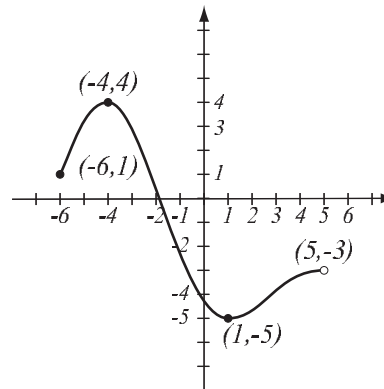
**3.** (6pts)  Solve the inequality. Write the solution in interval form.

$|x - 7| > 7$

**4.** (10pts)  Find the equation of the line (in form $y = mx + b$) whose $x$-intercept is 3 and passes through $(-3, 4)$. Is this line perpendicular to the line $3x - 2y = 7$? Draw both lines.

**5.** (10pts)  The graph of $f(x)$ is drawn below. Find the graphs of $-f(x)+1$ and $f(-2x)$ and label all the relevant points.



**6.** (12pts)  The quadratic function $f(x) = x^2 - 6x + 10$ is given. Do the following without using the calculator.
a) Find the $x$- and $y$-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

**7.** (8pts)  Find the domain of the function $f(x) = \dfrac{\ln(3x - 10)}{x^2 - 4x - 12}$ and write it in interval notation.

**8.** (6pts) Let $f(x) = \dfrac{2x-3}{x+1}$. Find the formula for $f^{-1}$.

**9.** (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{x^{-5}}{32\sqrt[4]{y^7}} =$$

**10.** (6pts) If you invest \$4,000 in an account bearing 3.05%, compounded quarterly, how much do you have in 10 years?

**11.** (14pts) The polynomial $f(x) = (x-3)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the $y$-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of $f$ (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

Solve the equations.

**12.** (8pts) $\sqrt{3x+13} + 5 = x$

**13.** (8pts) $4^{x-1} = 14^{3x-5}$

**14.** (14pts)  Greg drove one and a half hours from Paducah to Hopkinsville on a scenic route. On his way back, he took a route 9 miles shorter, so the trip lasted one hour and fifteen minutes. He drove at the same speed on both trips.
a) What was Greg's speed on both trips?
b) How long is the scenic route?

**15.** (14pts)  A developer has budgeted enough money to build 1500 feet of walls in a small retail building consisting of two stores with back rooms, where the front of the store is not walled in (see picture). The developer's goal is to maximize the total area of the building.
a) Express the area of the building as a function of the length of one of the sides $x$. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible area?
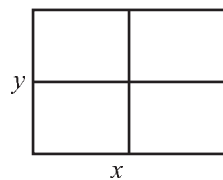
**16.** (12pts) The population of Spiriton was 72,000 in 1995 and 94,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.

a) Find $k$ and write the function that describes the population at time $t$ years since 1995. Graph it on paper.

b) Find the predicted population in the year 2017.

**Bonus.** (10pts) Recall that the distance between points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Consider the set of points $(x, y)$ that has equal distance to the point $(0, 2)$ and the $x$-axis (distance from a point to the $x$-axis is the $y$-coordinate of the point). Write the equation that a point $(x, y)$ with this property has to satisfy, and simplify it to form $y = f(x)$. You should get a parabola. (This is the classical definition of a parabola: set of points whose distance to a fixed point and a fixed line is equal.)