College Algebra — Final Exam
MAT 140, Spring 2016 — D. Ivanšić

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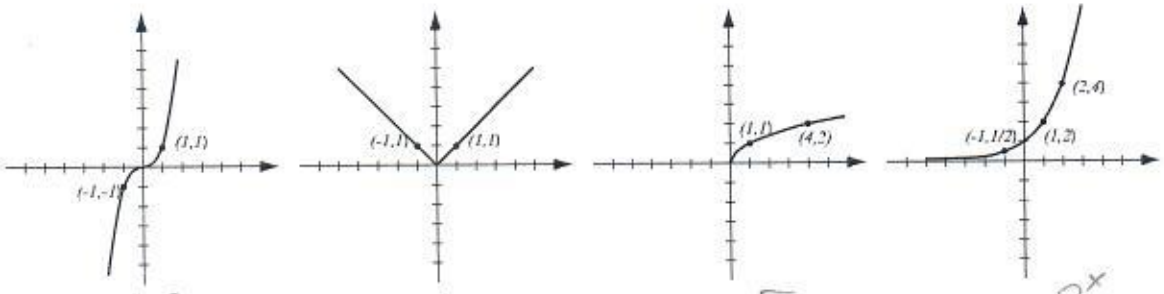
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = x^3$ $\quad y = |x| \quad y = \sqrt{x} \quad y = 2^x$

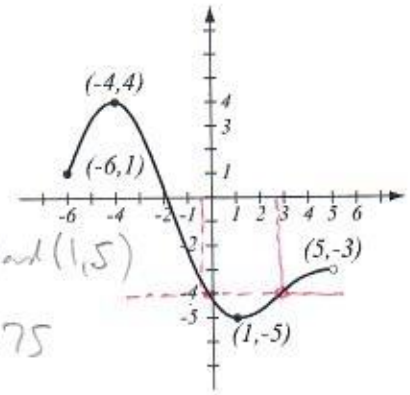
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) What is the domain of f? $[-6, 5)$

b) What is the range of f? $[-5, 4]$

c) Find the intervals of increase of f. $(-6, -4)$ and $(1, 5)$

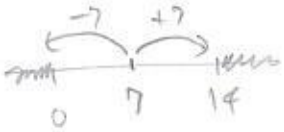
d) What are the solutions of the equation $f(x) = -4$? $-0.25,\ 2.75$



3. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 7| > 7$

distance from x to $7 > 7$ $\quad (-\infty, 0) \cup (14, \infty)$



4. (10pts) Find the equation of the line (in form $y = mx + b$) whose x-intercept is 3 and passes through $(-3, 4)$. Is this line perpendicular to the line $3x - 2y = 7$? Draw both lines.

Line passes through $(3, 0)$ and $(-3, 4)$

$m = \frac{4-0}{-3-3} = \frac{4}{-6} = -\frac{2}{3}$

$y - 0 = -\frac{2}{3}(x - 3)$

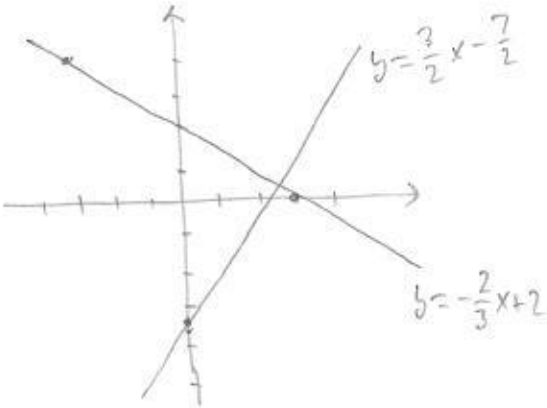
$y = -\frac{2}{3}x + 2$

$3x - 2y = 7$

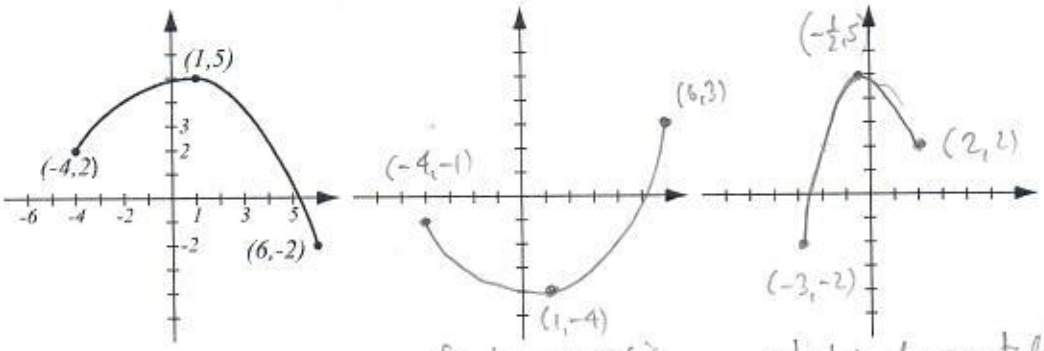
$3x - 7 = 2y$

$y = \frac{3}{2}x - \frac{7}{2}$

slope $= \frac{3}{2}$, which is opposite reciprocal of $-\frac{2}{3}$, so they are perpendicular



5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x)+1$ and $f(-2x)$ and label all the relevant points.



reflect in x-axis
shift up 1

stretch horizontally, factor = $\frac{1}{2}$
reflect in y-axis

6. (12pts) The quadratic function $f(x) = x^2 - 6x + 10$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = 10$

x-int: $x^2 - 6x + 10 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{-4}}{2}$$

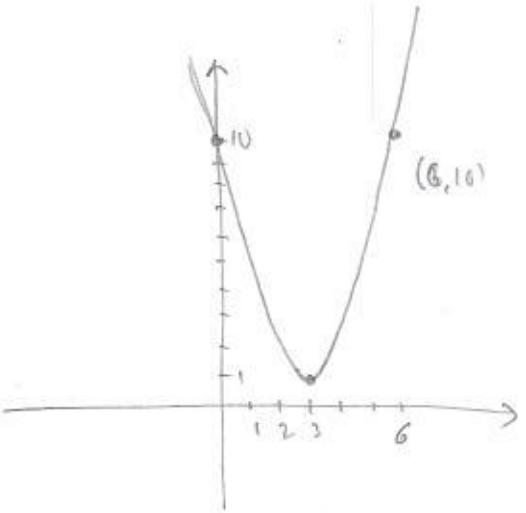
no real sol.

- no x-int.

b) $h = -\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$

$k = 3^2 - 6 \cdot 3 + 10$

$= 1$



7. (8pts) Find the domain of the function $f(x) = \dfrac{\ln(3x-10)}{x^2-4x-12}$ and write it in interval notation.

must have:

$3x - 10 > 0$

$3x > 10$

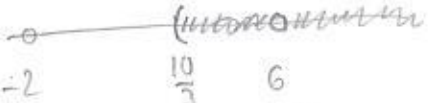
$x > \frac{10}{3}$

Can't have:

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, -2$



$(\frac{10}{3}, 6) \cup (6, \infty)$

8. (6pts) Let $f(x) = \dfrac{2x-3}{x+1}$. Find the formula for f^{-1}.

$y = \frac{2x-3}{x+1}$

$y(x+1) = 2x-3$

$yx + y = 2x-3$

$yx - 2x = -y-3$

$x(y-2) = -y-3$

$x = \frac{-y-3}{y-2} = \frac{y+3}{2-y}$

$f^{-1}(y) = \frac{y+3}{2-y}$

9. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_2 \dfrac{x^{-5}}{32\sqrt[4]{y^7}} = \log_2 x^{-5} - \log_2 32 - \log_2 y^{\frac{7}{4}}$

$= -5\log_2 x - 5 - \frac{7}{4}\log_2 y$

10. (6pts) If you invest \$4,000 in an account bearing 3.05%, compounded quarterly, how much do you have in 10 years?

$A = P\left(1+\frac{r}{n}\right)^{nt} = 4000\left(1+\frac{0.0305}{4}\right)^{4\cdot 10} = 4000 \cdot 1.007625^{40}$

$= 4000 \cdot 1.355\ldots$

$= 5420.23$

11. (14pts) The polynomial $f(x) = (x-3)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

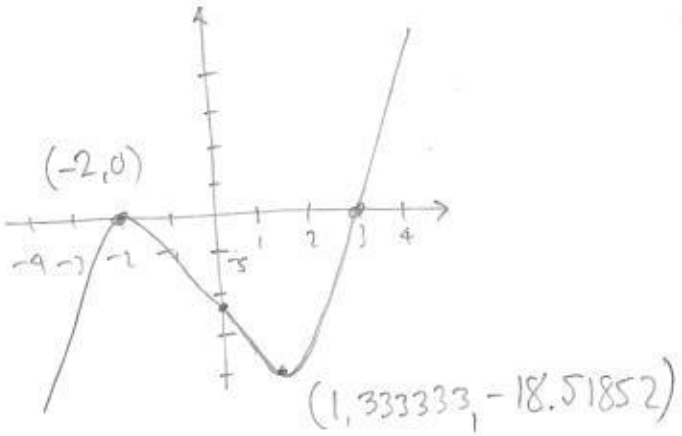
a) like $x \cdot x^2 = x^3$

b)

zeros	3	-2
mult.	1	2

y-int: $f(0) = -3 \cdot 2^2 = -12$

c)



d) $f(-2) = 0$ is a local max

$f(1.333333) = -18.51852$ is a local min.

Solve the equations.

12. (8pts) $\sqrt{3x+13} + 5 = x$

$\sqrt{3x+13} = x-5 \quad |^2$

$3x+13 = x^2 - 10x + 25$

$x^2 - 13x + 12 = 0$

$(x-12)(x-1) = 0$

$x = 1$ or 12

Check. $\sqrt{36+13} + 5 \stackrel{?}{=} 12$

$7+5 = 12$ yes

$\sqrt{3+13} + 5 \stackrel{?}{=} 1$

$4+5 \stackrel{?}{=} 1$ no

13. (8pts) $4^{x-1} = 14^{3x-5} \quad | \ln$

$\ln 4^{x-1} = \ln 14^{3x-5}$

$(x-1)\ln 4 = (3x-5)\ln 14$

$x\ln 4 - \ln 4 = 3x\ln 14 - 5\ln 14$

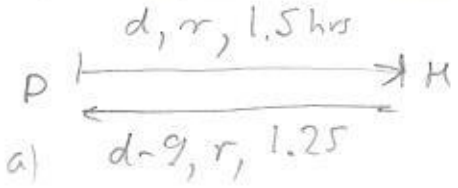
$x\ln 4 - 3x\ln 14 = \ln 4 - 5\ln 14$

$x(\ln 4 - 3\ln 14) = \ln 4 - 5\ln 14$

$x = \dfrac{\ln 4 - 5\ln 14}{\ln 4 - 3\ln 14} = \dfrac{5\ln 14 - \ln 4}{3\ln 14 - \ln 4}$

$= 1.808178$

14. (14pts) Greg drove one and a half hours from Paducah to Hopkinsville on a scenic route. On his way back, he took a route 9 miles shorter, so the trip lasted one hour and fifteen minutes. He drove at the same speed on both trips.
a) What was Greg's speed on both trips?
b) How long is the scenic route?



a) $d = r \cdot 1.5$

$d - 9 = r \cdot 1.25$

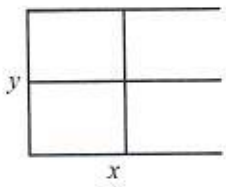
$1.5r - 9 = 1.25r$

$0.25r = 9 \quad | \cdot 4$

$r = 36$ mph

b) $36 \cdot 1.5 = 54$ miles

15. (14pts) A developer has budgeted enough money to build 1500 feet of walls in a small retail building consisting of two stores with back rooms, where the front of the store is not walled in (see picture). The developer's goal is to maximize the total area of the building.
a) Express the area of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible area?



$3x + 2y = 1500$

$2y = 1500 - 3x$

$y = 750 - \frac{3}{2}x$

$A = xy = x\left(750 - \frac{3}{2}x\right)$

$= -\frac{3}{2}x^2 + 750x$

Domain:
must have:

$x \geq 0$

$y \geq 0$

$750 - \frac{3}{2}x \geq 0$

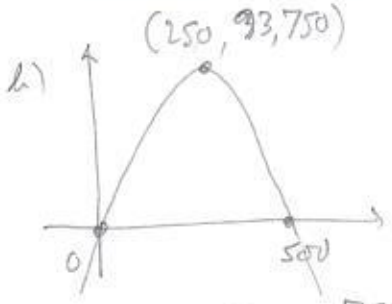
$750 \geq \frac{3}{2}x \quad | \cdot \frac{2}{3}$

$x \leq 500$

Domain

$[0, 500]$

b)



Vertex: $h = -\frac{b}{2a} = -\frac{750}{2 \cdot (-\frac{3}{2})} = \frac{750}{3} = 250$

$k = 250 \cdot 375 = 93{,}750$

Dimensions are 250×375

Max area is 93,750 sq. feet.

16. (12pts) The population of Spiriton was 72,000 in 1995 and 94,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1995. Graph it on paper.
b) Find the predicted population in the year 2017.

a) $P(t) = 72\,e^{k\cdot t}$ (in thousands)

$94 = 72\,e^{k\cdot 15}$ | $\div 72$

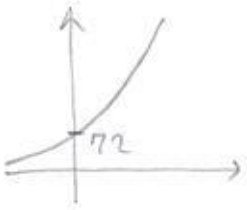
$\frac{94}{72} = e^{k\cdot 15}$ | $\ln$

$\ln\frac{47}{36} = k\cdot 15$

$k = \frac{\ln\frac{47}{36}}{15} = 0.0177752$

$P(t) = 72\,e^{0.0177752\,t}$

b)

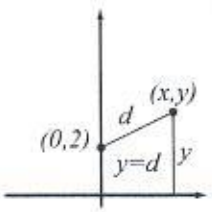


$P(22) = 72\,e^{0.0177752\cdot 22}$

$= 106.4549075$

About 106,455 people in 2017.

Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Consider the set of points (x, y) that has equal distance to the point $(0, 2)$ and the x-axis (distance from a point to the x-axis is the y-coordinate of the point). Write the equation that a point (x, y) with this property has to satisfy, and simplify it to form $y = f(x)$. You should get a parabola. (This is the classical definition of a parabola: set of points whose distance to a fixed point and a fixed line is equal.)



$d = \sqrt{(x-0)^2 + (y-2)^2}$

$d = y$ says

$\sqrt{x^2 + (y-2)^2} = y$ | ^{2}

$x^2 + (y-2)^2 = y^2$

$x^2 + y^2 - 4y + 4 = y^2$

$x^2 + 4 = 4y$

$y = \frac{x^2}{4} + 1$