College Algebra — Exam 4
MAT 140, Spring 2016 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_8 64 = 2$ $\quad 8^? = 64$

$\log_3 \frac{1}{81} = -4$ $\quad 3^? = \frac{1}{81} = \frac{1}{3^4} = 3^{-4}$

$\log_a \sqrt[5]{a^2} = \frac{2}{5}$ $\quad a^? = \sqrt[5]{a^2} = a^{\frac{2}{5}}$

$\log_{b^4} \sqrt{b} = \frac{1}{8}$ $\quad (b^4)^? = \sqrt{b} = b^{\frac{1}{2}} \quad ? = \frac{1}{8}$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_5 6$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_5 6 = \frac{\ln 6}{\ln 5} = 1.113283$

3. (5pts) If $\log_a 3 = c$ and $\log_a 7 = d$, express in terms of c and d:

$\log_a 21 = \log_a(3 \cdot 7)$
$= \log_a 3 + \log_a 7$
$= c + d$

$\log_a \frac{27}{49} = \log_a \frac{3^3}{7^2} = \log_a 3^3 - \log_a 7^2$
$= 3\log_a 3 - 2\log_a 7$
$= 3c - 2d$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

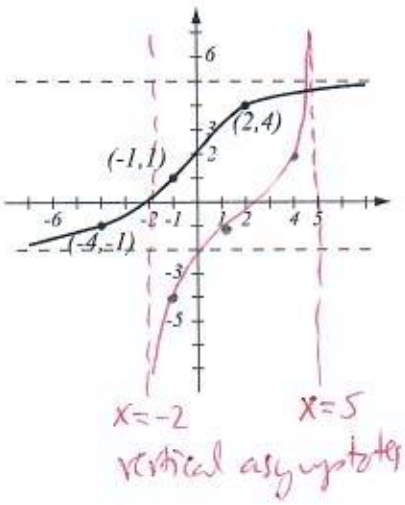
$\ln \frac{e^2 x^3}{\sqrt[3]{y^5}} = \ln e^2 + \ln x^3 - \ln y^{\frac{5}{3}}$
$= 2 + 3\ln x - \frac{5}{3}\ln y$

5. (12pts) Write as a single logarithm. Simplify if possible.

$2\log_4(x^2y^{-4}) + 3\log_4(x^{-2}y^3) = \log_4 (x^2y^{-4})^2 + \log_4 (x^{-2}y^3)^3 = \log_4(x^4y^{-8}) + \log_4(x^{-6}y^9)$
$= \log_4(x^4y^{-8} \cdot x^{-6}y^9) = \log_4(x^{-2}y) = \log_4 \frac{y}{x^2}$

$3\log(x+7) - 4\log(x^2+4x-21) + 5\log(x-3) = \log(x+7)^3 - \log\left((x-3)(x+7)\right)^4 + \log(x-3)^5$
$= \log \frac{(x+7)^3(x-3)^5}{(x-3)^4(x+7)^4} = \log \frac{x-3}{x+7}$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) yes - it passes the horizontal line test

7. (9pts) Let $f(x) = \dfrac{3-x}{x+2}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \dfrac{3-x}{x+2}$

$(x+2)y = 3-x$

$yx+2y = 3-x$

$yx+x = 3-2y$

$x(y+1) = 3-2y$

$x = \dfrac{3-2y}{y+1} = f^{-1}(y)$

b) Range of f = domain of f^{-1}

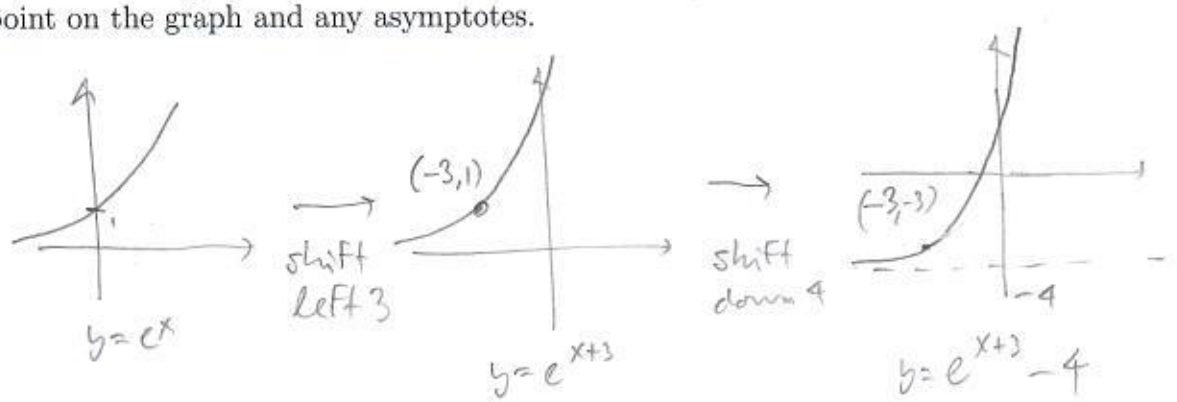
can't have $y+1=0$

$y=-1$

$\{y \mid y \neq -1\}$

$= (-\infty,-1) \cup (-1,\infty)$

8. (6pts) Using transformations, draw the graph of $f(x) = e^{x+3} - 4$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



9. (6pts) Find the domain of the function $f(x) = \dfrac{\log_6(4x-15)}{x-4}$ and write it in interval notation.

Must have: $4x-15>0$ Can't have $x-4=0$

$4x>15$ $x=4$ $\left(\frac{15}{4},4\right)\cup(4,\infty)$

$x>\frac{15}{4}$

15/4 4

10. (8pts) How much should you invest in an account bearing 3.66%, compounded monthly, if you wish to have \$3,000 in five years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3000 = P\left(1+\frac{0.0366}{12}\right)^{12\cdot 5}$

$3000 = P, 1.2004..$

$P = \frac{3000}{1.2004..} = 2499.00$

Solve the equations.

11. (8pts) $7^{2x-1} = 5^{x+2}$ $\ln$

$\ln 7^{2x-1} = \ln 5^{x+2}$

$(2x-1)\ln 7 = (x+2)\ln 5$

$2x\ln 7 - \ln 7 = x\ln 5 + 2\ln 5$

$2x\ln 7 - x\ln 5 = \ln 7 + 2\ln 5$

$x(2\ln 7 - \ln 5) = \ln 7 + 2\ln 5$

$x = \dfrac{2\ln 5 + \ln 7}{2\ln 7 - \ln 5} = 2.262895$

12. (10pts) $3^{2x} - 6\cdot 3^x = 18$

Let $u = 3^x$, then $3^{2x} = (3^x)^2 = u^2$

$u^2 - 6u = 18$

$u^2 - 6u - 18 = 0$

$u = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4\cdot 1\cdot(-18)}}{2\cdot 1}$

$= \dfrac{6 \pm \sqrt{36+72}}{2}$

$= \dfrac{6 \pm \sqrt{108}}{2} = \dfrac{6 \pm 2\sqrt{27}}{2} = 3 \pm 3\sqrt{3}$ (4·27)

$3^x = 3+3\sqrt{3}$ or $3^x = \underbrace{3-3\sqrt{3}}_{<0}$

$\ln 3^x = \ln(3+3\sqrt{3})$ so no solution

$x\ln 3 = \ln(3+3\sqrt{3})$

$x = \dfrac{\ln(3+3\sqrt{3})}{\ln 3}$

13. (12pts) The population of Spiriton was 95,000 in 2000 and 126,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2021.

a) $P(t) = 95e^{kt}$ (in thousands)

$P(10) = 126$

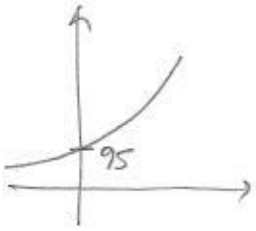
$126 = 95e^{k \cdot 10} \quad | \div 95$

$\frac{126}{95} = e^{k \cdot 10} \quad | \ln$

$\ln \frac{126}{95} = \ln e^{k \cdot 10}$

$\ln \frac{126}{95} = k \cdot 10$

$k = \frac{\ln \frac{126}{95}}{10} = 0.0282405$



b) Need $P(21) = 95e^{0.0282405 \cdot 21}$

$= 171.902495,$

so 171,902 people

Bonus (10pts) What is better: depositing money into an account with interest rate 4.5%, compounded quarterly, or into an account with interest rate 4.4%, compounded monthly? (To determine this, calculate the amount at the end of 1 year, if \$100 is deposited into either account.)

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

4.5%, comp. quarterly

$100 \cdot \left(1 + \frac{0.045}{4}\right)^{4 \cdot 1}$

$= 104.58$

4.4%, comp. monthly

$100\left(1 + \frac{0.044}{12}\right)^{12 \cdot 1}$

$= 104.49$

More money, so better.