College Algebra — Exam 3
MAT 140, Spring 2016 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $i(i+1) - 2i(i-1) = i^2 + i - 2i^2 + 2i$

$= -1 + 3i + 2 = 1 + 3i$

2. (6pts) $\dfrac{4-i}{2+5i} = \dfrac{4-i}{2+5i} \cdot \dfrac{2-5i}{2-5i} = \dfrac{8-2i-20i+5i^2}{2^2-(5i)^2} = \dfrac{8-22i-5}{4-25i^2} = \dfrac{3-22i}{29}$

$+25$

3. (4pts) Simplify and justify your answer.

$i^{114} = (i^2)^{57} = (-1)^{57} = -1$

4. (6pts) Solve the equation by completing the square.

$x^2 + 12x + 40 = 0 \quad | + 6^2$

$x^2 + 2 \cdot x \cdot 6 + 6^2 + 40 = 6^2 \quad | -40$

$(x+6)^2 = 36 - 40$

$(x+6)^2 = -4$

$x + 6 = \pm\sqrt{-4} = \pm 2i$

$x = -6 \pm 2i$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x+9| \geq 2$

$|x - (-9)| \geq 2$

dist. from x to $-9 \geq 2$

$-2 \quad +2$

$-11 \quad -9 \quad -7$

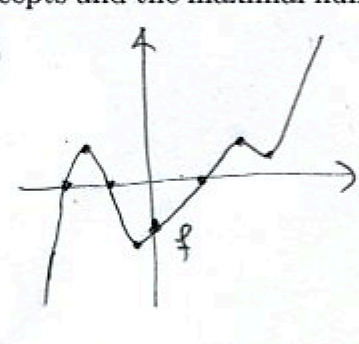
$(-\infty, -11] \cup [-7, \infty)$

6. (6pts) Let $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, and suppose $a > 0$ and $f < 0$.
a) State the maximum number of x-intercepts that the graph of P may have.
b) State the maximum number of turning points that the graph of P may have.
c) Draw a possible graph of P, if it has three x-intercepts and the maximal number of turning points.

a) degree $= 5$, so at most 5 x-int

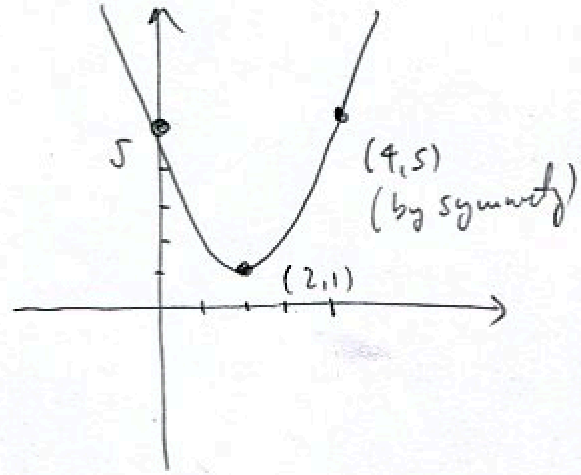
b) at most $5 - 1 = 4$ turning pts

c)



behaves like ax^5

7. (12pts) The quadratic function $f(x) = x^2 - 4x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.



y-int: $f(0) = 5$

x-int: $x^2 - 4x + 5 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 5}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$ no real sol- no x-int.

b) $h = -\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$ vertex: $(2,1)$

$k = f(2) = 4 - 8 + 5 = 1$

Solve the equations:

8. (8pts) $\dfrac{x^2 - x}{x^2 - 9} + \dfrac{4}{x+3} = \dfrac{1}{x-3}$ $\Big| \cdot (x-3)(x+3)$

$(x-3)(x+3)$

$x^2 - x + 4(x-3) = x + 3$

$x^2 - x + 4x - 12 = x + 3$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5, 3$

3 is not a sol. (gives 0 in denom.)

$x = -5$

9. (8pts) $\sqrt{6x+7} - x = 2$

$\sqrt{6x+7} = x + 2 \quad |^2$

$6x + 7 = x^2 + 4x + 4 \quad | -6x - 7$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

Check: $x = 3$ $\sqrt{18+7} - 3 \stackrel{?}{=} 2$

$5 - 3 \stackrel{?}{=} 2$ yes

$x = -1$ $\sqrt{-6+7} - (-1) \stackrel{?}{=} 2$

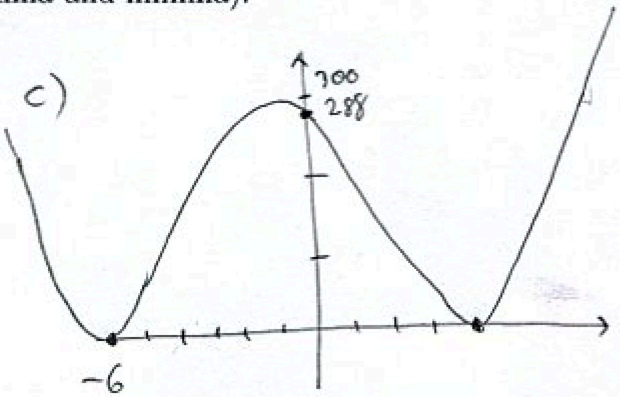
$\sqrt{1} + 1 \stackrel{?}{=} 2$ yes

Sol: $-1, 3$.

10. (14pts) The polynomial $f(x) = \frac{1}{2}(x-4)^2(x+6)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) leading term is
$\frac{1}{2}x^2 \cdot x^2 = \frac{1}{2}x^4$
Behaves like $\frac{1}{2}x^4$

c)



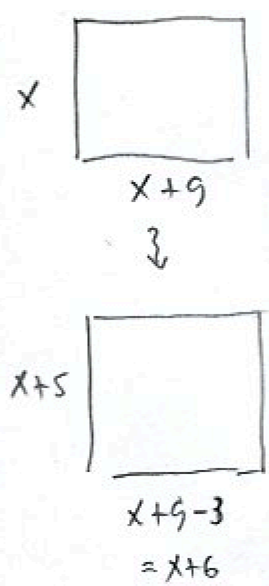
b)

zero	4	−6
mult.	2	2

y-int: $f(0) = \frac{1}{2}(-4)^2 \cdot 6^2$
$= 8 \cdot 36 = 288$

d) local min: $f(-6) = 0$
$f(4) = 0$
local max: $f(-1) = 312.5$

11. (12pts) In a rectangle, one side is 9cm longer than the other. If the shorter side is lengthened by 5cm, and the longer side shortened by 3cm, we get a rectangle with twice the area of the original one. What are the dimensions of the original rectangle?



has twice the area

$2x(x+9) = (x+5)(x+6)$

$2x^2 + 18x = x^2 + 11x + 30$

$x^2 + 7x - 30 = 0$

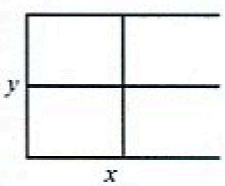
$(x+10)(x-3) = 0$

$x = -10, 3$

doesn't fit context, $x \geq 0$

Original rectangle is
3×12
$x \quad x+9$

12. (14pts) A developer has budgeted enough money to build 1200 feet of walls in a small retail building consisting of two stores with back rooms, where the front of the store is not walled in (see picture). The developer's goal is to maximize the total area of the building.
a) Express the area of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible area?



$\underbrace{3x+2y}_{\text{length of walls}} = 1200$

$2y = 1200 - 3x$

$y = 600 - \frac{3}{2}x$

a)

$A = xy = x\left(600 - \frac{3}{2}x\right) = -\frac{3}{2}x^2 + 600x$

Domain:

Must have: $x \geq 0$

$y \geq 0$

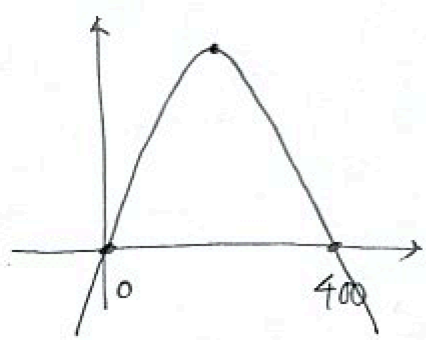
$0 \leq x \leq 400$

$600 - \frac{3}{2}x \geq 0$

$\frac{3}{2}x \leq 600$

$x \leq 400$

b)



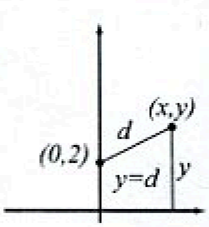
Vertex is $h = -\frac{b}{2a} = -\frac{600}{2\cdot(-\frac{3}{2})} = 200$

$A(200) = 200 \cdot 300 = 6\,000$

The rectangle 200×300 maximizes the area

Max area is 6000 ft^2

Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$. Consider the set of points (x, y) that has equal distance to the point $(0, 2)$ and the x-axis (distance from a point to the x-axis is the y-coordinate of the point). Write the equation that a point (x, y) with this property has to satisfy, and simplify it to form $y = f(x)$. You should get a parabola. (This is the classical definition of a parabola: set of points whose distance to a fixed point and a fixed line is equal.)



Condition is

$d = y$

distance from $(0,2)$ to $(x,y) = y$

$\sqrt{(x-0)^2 + (y-2)^2} = y \quad |^2$

$x^2 + (y-2)^2 = y^2$

$x^2 + y^2 - 4y + 4 = y^2 \quad |-y^2$

$x^2 - 4y + 4 = 0$ solve for y

$4y = x^2 + 4$

$y = \frac{1}{4}x^2 + 1$, which is a parabola