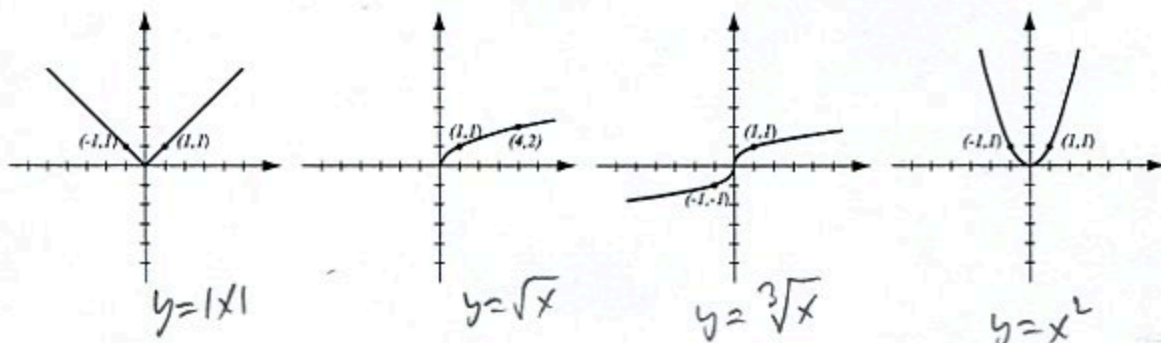


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (14pts) First find the equation of the line (in form $y = mx + b$) whose x -intercept is 3 and y -intercept is 2. Then find the equation of the line that is perpendicular to the first one and passes through the origin. Draw both lines.

Passes through
 $(3, 0)$ and $(0, 2)$

$$m = \frac{2-0}{0-3} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + 2$$

\uparrow
 y -int.

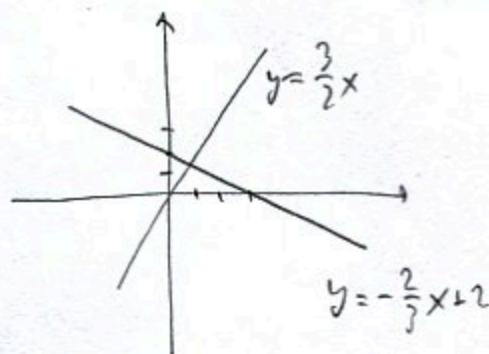
Slope of perp. line

$$\text{is } m = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}$$

Equation of perp. line
(passes through $(0, 0)$)

$$y - 0 = \frac{3}{2}(x - 0)$$

$$y = \frac{3}{2}x$$



3. (5pts) Solve the inequality and write your solution in interval notation.

$$2x + 1 \leq 7 \text{ or } 3x - 6 > 8$$

$$2x \leq 6 \quad 3x > 14$$

$$x \leq 3 \text{ or } x > \frac{14}{3}$$

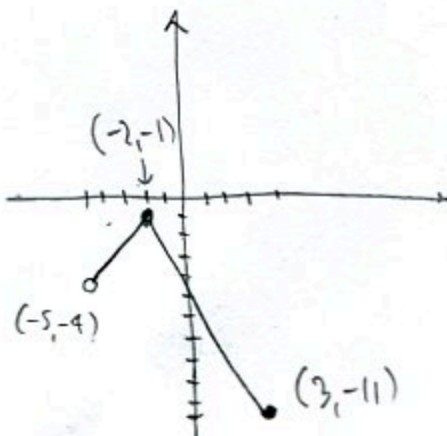
~~$x \leq 3$~~ ~~$x > \frac{14}{3}$~~

$$(-\infty, 3] \cup \left(\frac{14}{3}, \infty\right)$$

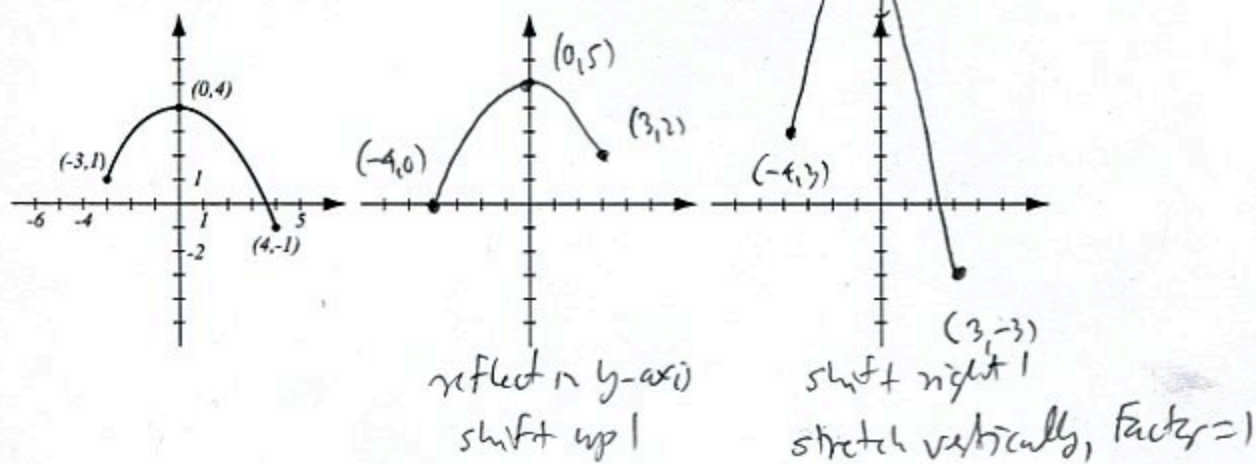
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x + 1, & \text{if } -5 < x \leq -2 \\ -5 - 2x, & \text{if } -2 < x \leq 3 \end{cases}$$

x	$x+1$	x	$-5-2x$
-5	-4	-2	-1
-2	-1	3	-11



5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) + 1$ and $3f(x+1)$ and label all the relevant points.



6. (14pts) Let $f(x) = x^2 + 1$, $g(x) = \frac{2}{\sqrt{3x+1}}$.

Find the following (simplify where possible):

$$\begin{aligned}(f+g)(5) &= f(5) + g(5) \\ &= (5^2 + 1) + \frac{2}{\sqrt{3 \cdot 5 + 1}} \\ &= 26 + \frac{2}{4} = 26.5\end{aligned}$$

$$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) = g((-2)^2 + 1) \\ &= g(5) = \frac{2}{\sqrt{15+1}} = \frac{2}{4} = \frac{1}{2}\end{aligned}$$

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) = (x^2 + 1) \cdot \frac{2}{\sqrt{3x+1}} \\ &= \frac{2(x^2 + 1)}{\sqrt{3x+1}}\end{aligned}$$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f\left(\frac{2}{\sqrt{3x+1}}\right) \\ &= \left(\frac{2}{\sqrt{3x+1}}\right)^2 + 1 = \frac{4}{3x+1} + 1 = \frac{4+3x+1}{3x+1} \\ &= \frac{3x+5}{3x+1}\end{aligned}$$

The domain of g in interval notation

Must have $3x+1 > 0$ (not ≥ 0 since could have 0 in denom.)

$$\begin{aligned}3x &> -1 \\ x &> -\frac{1}{3}\end{aligned}$$

$\left(-\frac{1}{3}, \infty\right)$

7. (3pts) Consider the function $h(x) = \frac{1}{x^3 - 17x}$. Find functions f and g , neither of which is the "stupid" one, so that $h(x) = f(g(x))$.

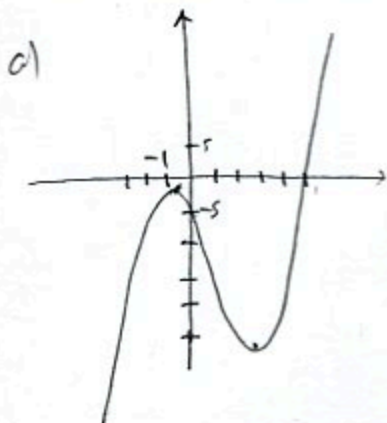
$$\begin{aligned}g(x) &= x^3 - 17x \\ f(x) &= \frac{1}{x}\end{aligned}$$

8. (12pts) Let $f(x) = x^3 - 3x^2 - 7x - 4$ (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.

b) Find the local maxima and minima for this function.

c) State the intervals where the function is increasing and where it is decreasing.



b) Local min is $f(2.825742) = -25.17161$

Local max is $f(-0.825741) = -0.828388$

c) Increasing on $(-\infty, -0.825741)$ and $(2.825742, \infty)$

Decreasing on $(-0.825741, 2.825742)$

9. (12pts) Fiona is considering her options for a cab ride: the company Quicky Qabs charges a \$5 startup fee, which includes three miles of travel, and \$2 per mile after three miles, and the company Speedy Sedan charges a \$7 startup fee that includes five miles of travel, and \$2.50 per mile after five miles. If Fiona will travel at least five miles, for which length of travel is Speedy Sedan better?

$x = \text{no. of miles traveled, } x \geq 5$

QQ cost $5 + 2(x-3)$

$0.5x \leq 4.5$

SS cost $7 + 2.50(x-5)$

$x \leq 9$

cost of SS \leq cost of QQ

For trips between 5 and 9 miles,
SS is better.

$7 + 2.50(x-5) \leq 5 + 2(x-3)$

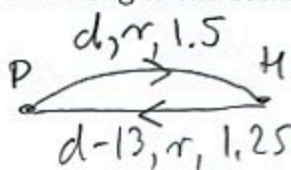
$7 + 2.5x - 12.5 \leq 5 + 2x - 6$

$0.5x + 5.5 \leq -1$

10. (14pts) Greg drove one and a half hours from Paducah to Hopkinsville on a scenic route. On his way back, he took a route 13 miles shorter, so the trip lasted one hour and fifteen minutes. He drove at the same speed on both trips.

a) What was Greg's speed on both trips?

b) How long is the scenic route?



$r = \text{Greg's speed}$

$$r = \frac{13}{0.25} = 52 \text{ mph}$$

$$d = r \cdot 1.5$$

a) Greg drove 52 mph

$$d - 13 = r \cdot 1.25$$

b) $52 \cdot 1.5 = 78 \text{ miles}$

$$r \cdot 1.5 - 13 = r \cdot 1.25$$

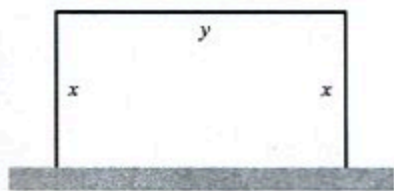
$$1.5r - 13 = 1.25r$$

$$0.25r = 13$$

Bonus. (14pts) Eric wishes to enclose a rectangular pen for his dog along a wall of his house (there is no fence along the wall). The pen is to have area of 500 square feet and Eric wishes to minimize the length of fence used.

a) Express the length of the fence used as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the length function in order to find the minimum. What are the dimensions of the pen that give the minimum fence length?

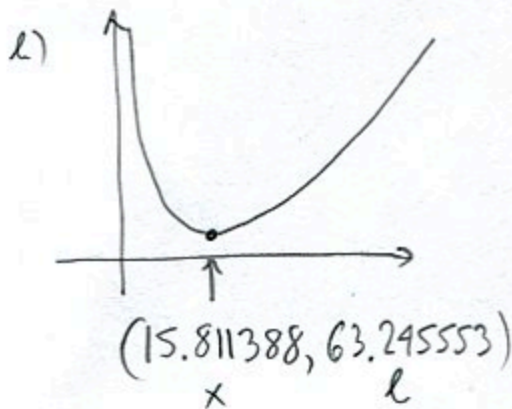


$$a) \quad x \cdot y = 500, \quad y = \frac{500}{x}$$

$$l = 2x + y$$

$$l = 2x + \frac{500}{x}$$

Domain: $x > 0$



dimensions are $15.811388 \times 31.622777$

$$= \frac{500}{x}$$