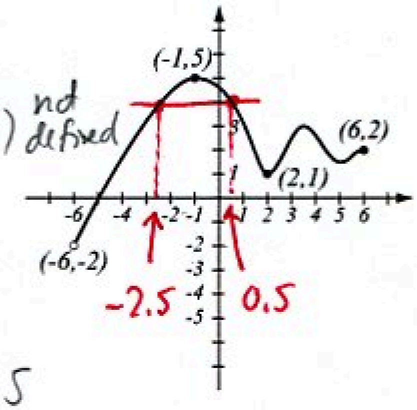
College Algebra — Exam 1 **MAT 140, Spring 2016 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(-1)$ and $f(-6)$. $f(-1)=5$, $f(-6)$ nd defined

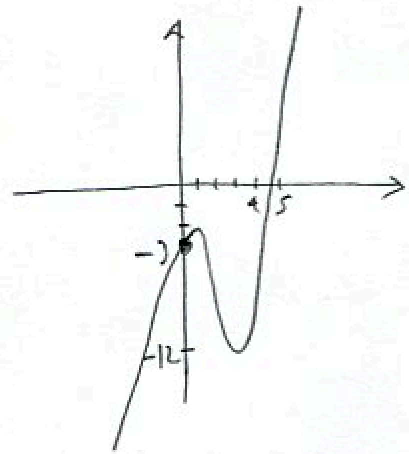
b) What is the domain of f? $(-6,6]$

c) What is the range of f? $(-2,5]$

d) What are the solutions of the equation $f(x)=4$?

y-coord = 4 x = −2.5, 0.5

2. (10pts) Use your calculator to accurately sketch the graph of $y=x^3-6x^2+6x-3$. Draw the graph here, and indicate units on the axes. Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).



y-int: −3

x-int: 4.900572

3. (4pts) Convert to scientific notation or a decimal number:

$4.171824\times10^6 =$ 4,171,824

$0.0007459 =$ 7.459×10^{-4}

Use formulas to expand:

4. (3pts) $(x-y^4)(x+y^4)=$ $x^2-(y^4)^2 = x^2-y^8$

5. (4pts) $(3s+5t)^2=$ $(3s)^2+2\cdot3s\cdot5t+(5t)^2 = 9s^2+30st+25t^2$

6. (5pts) Factor: $u^3+27=$ $u^3+3^3=(u+3)(u^2-3u+3^2)$

$=(u+3)(u^2-3u+9)$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

7. (2pts) $\sqrt{48} = \sqrt{16 \cdot 3}$

$= 4\sqrt{3}$

8. (5pts) $\sqrt{125x^6y^3} =$

$= \sqrt{25 \cdot 5 \cdot (x^3)^2 y^2 \cdot y} = 5\sqrt{5}\,|x^3|\,|y|\sqrt{y}$

$= 5\,|x^3 y|\sqrt{5y}$

9. (8pts) Simplify.

$\frac{x-1}{x^2-9} - \frac{4x}{x^2-4x-21} = \frac{x-1}{(x+3)(x-3)} - \frac{4x}{(x-7)(x+3)} =$

$= \frac{(x-1)(x-7) - 4x(x-3)}{(x+3)(x-3)(x-7)} = \frac{x^2-8x+7-4x^2+12x}{(x+3)(x-3)(x-7)}$

$= \frac{-3x^2+4x+7}{(x+3)(x-3)(x-7)}$

prod = −21   7, −3 $-3x^2-3x+7x+7$

sum = 4 $= -3x(x+1)+7(x+1)$

$= (-3x+7)(x+1)$

$= \frac{(-3x+7)(x+1)}{(x+3)(x-3)(x-7)}$

10. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$\frac{\left(x^9y^{-\frac{3}{2}}\right)^{\frac{1}{3}}}{\left(x^{\frac{1}{2}}y^{\frac{3}{2}}\right)^5} = \frac{x^{9\cdot\frac{1}{3}}\, y^{-\frac{3}{2}\cdot\frac{1}{3}}}{x^{\frac{1}{2}\cdot 5}\, y^{\frac{3}{2}\cdot 5}} = \frac{x^3\, y^{-\frac{1}{2}}}{x^{\frac{5}{2}}\, y^{\frac{15}{2}}} = x^{3-\frac{5}{2}}\, y^{-\frac{1}{2}-\frac{15}{2}} = x^{\frac{1}{2}} y^{-8}$

$= \frac{x^{\frac{1}{2}}}{y^8} = \frac{\sqrt{x}}{y^8}$

11. (6pts) Rationalize the denominator.

$\frac{2\sqrt{3}-5}{4+\sqrt{3}} \cdot \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{8\sqrt{3}-20-2\sqrt{3}^2+5\sqrt{3}}{4^2-\sqrt{3}^2} = \frac{13\sqrt{3}-20-6}{16-3} = \frac{-26+13\sqrt{3}}{13}$

$= -2+\sqrt{3}$

12. (5pts) Solve the equation for t.

$ct - 5a = t + 1$

$ct - t = 5a + 1$

$t(c-1) = 5a + 1$

$t = \frac{5a+1}{c-1}$

13. (8pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \frac{x - 13}{x^2 + 6x - 40}$

Can't have $x^2 + 6x - 40 = 0$

$(x+10)(x-4) = 0$

$x = -10, 4$

$-10 \quad 4$

$(-\infty, -10) \cup (-10, 4) \cup (4, \infty)$

$g(x) = \sqrt[3]{3x - 11}$

$\sqrt[3]{x}$ is always defined,

so domain is all reals,

$(-\infty, \infty)$

14. (9pts) Let $g(x) = 2x^2 + 3x - 7$. Find the following (simplify where appropriate).

$g(-2) = 2\cdot(-2)^2 + 3\cdot(-2) - 7$

$= 8 - 6 - 7 = -5$

$g(-u) = 2(-u)^2 + 3(-u) - 7$

$= 2u^2 - 3u - 7$

$g(\sqrt{x+5}) = 2(\sqrt{x+5})^2 + 3\sqrt{x+5} - 7$

$= 2(x+5) + 3\sqrt{x+5} - 7$

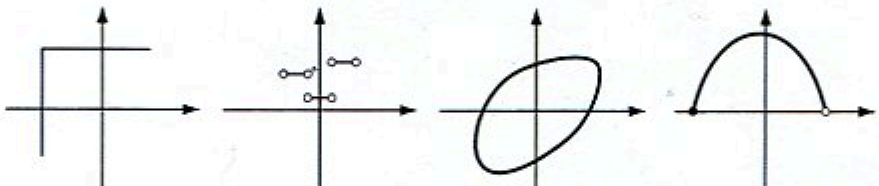
$= 2x + 3 + 3\sqrt{x+5}$

$g(x+5) = 2(x+5)^2 + 3(x+5) - 7$

$= 2(x^2 + 10x + 25) + 3x + 15 - 7$

$= 2x^2 + 23x + 58$

15. (5pts) Which of the folowing graphs are graphs of functions? Why?



no yes no yes

The yeses pass the vertical line test

16. (10pts) The diameter of a circle has endpoints $(-3, -2)$ and $(1, 4)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

a) Center = midpoint of $(-3,-2)$ and $(1,4)$

$$= \left(\frac{-3+1}{2}, \frac{-2+4}{2}\right)$$

$$= (-1, 1)$$

r = distance from $(-3,-2)$ to $(-1,1)$

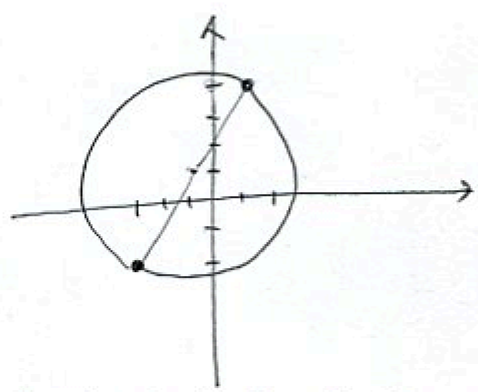
$$= \sqrt{(-1-(-3))^2 + (1-(-2))^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{13}$$

Equation of circle:

$$(x-(-1))^2 + (y-1)^2 = \sqrt{13}^2$$

$$(x+1)^2 + (y-1)^2 = 13$$



Bonus (10pts) Find the coordinates (x, y) of at least 4 points in the plane that lie on the curve with the equation $(x-2)^2 + (y+4)^2 = 10$. *(Hint: set one variable, and solve for the other; or draw the curve and infer some points from the picture.)*

One way:

Set $x = 1$

$$(1-2)^2 + (y+4)^2 = 10$$

$$(y+4)^2 = 9$$

$$y+4 = \pm 3$$

$$y = -7, -1$$

Set $x = 4$

$$(4-2)^2 + (y+4)^2 = 10$$

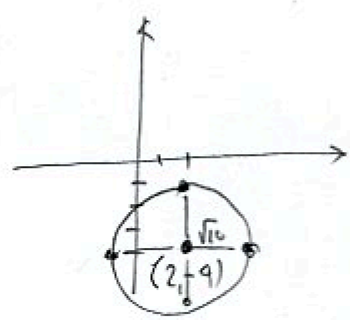
$$(y+4)^2 = 6$$

$$y+4 = \pm\sqrt{6}$$

$$y = -4 \pm \sqrt{6}$$

Points $(1,-1)$, $(1,-7)$, $(4, -4+\sqrt{6})$, $(4, -4-\sqrt{6})$

Other way: It's a circle with center $(2,-4)$, radius $\sqrt{10}$



Points are

$(2-\sqrt{10}, -4)$
$(2+\sqrt{10}, -4)$
$(2, -4+\sqrt{10})$
$(2, -4-\sqrt{10})$