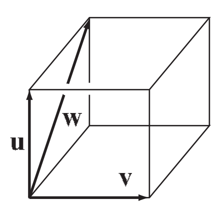
Calculus 3 — Exam 1	**Name:**
MAT 309, Fall 2016 — D. Ivanšić	*Show all your work!*

1. (13pts) Let $\mathbf{u} = \langle 2, 3, -5 \rangle$ and $\mathbf{v} = \langle 0, -3, 1 \rangle$.
a) Calculate $2\mathbf{u}$, $\mathbf{u} - 2\mathbf{v}$, and $\mathbf{u} \cdot \mathbf{v}$.
b) Find the unit vector in direction of $\mathbf{v}$.
c) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. (13pts) In the picture, vectors $\mathbf{u}$ and $\mathbf{v}$ are on sides of a cube with side-length 1, and $\mathbf{w}$ is the diagonal of one of the sides. Draw the vectors $\mathbf{u} \times \mathbf{w}$ and $\mathbf{v} \times \mathbf{w}$ and determine their length.



3. (8pts) Draw the region in $\mathbf{R}^3$ described by:

$y + z = 3$, $x \geq 0$

4. (12pts) Find the equation of the plane that contains the line $x = 3t$, $y = 7 - 2t$, $z = -4 + 2t$ and the point $(-2, 3, -4)$.

5. (16pts) This problem is about the surface $x + y^2 + 3z^2 = 0$.
a) Identify and sketch the intersections of this surface with the coordinate planes.
b) Sketch the surface in 3D, with coordinate system visible.

6. (22pts) The curve $\mathbf{r}(t) = \langle \sin(8t), 2\cos t, 2\sin t\rangle$ is given, $0 \leq t \leq 4\pi$.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = \pi/3$ and sketch the tangent line.
c) Set up the integral for the length of the curve. Simplify the function inside the integral as much as possible, but do not evaluate the integral.

7. (16pts) Consider the triangle whose vertices are intersections of the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{1} = 1$ with the coordinate axes.

a) Draw the triangle.

b) Find the area of the triangle.

c) Is this a right triangle?

Bonus (10pts) Standing at point $(2, 1, 1)$, you throw a rock with initial velocity vector $\mathbf{v_0} = \langle -1, 2, 8 \rangle$. Assuming gravity (let $g = 10$ here) acts in the usual negative z-direction, find the point where the rock hits the incline represented by the plane $3x + 4y + z - 5 = 0$.

Calculus 3 — Exam 2 **MAT 309, Fall 2016 — D. Ivanšić**

Name: ____________________

Show all your work!

1. (22pts) Let $h(x, y) = \sqrt{x} + y$.
a) Find the domain of h.
b) Sketch the contour map for the function, drawing level curves for levels $k = -2, -1, 0, 1, 2$. Note the domain on the picture.
c) At point $(9, -2)$, find the directional derivative of f in the direction of $\langle 1, -2 \rangle$. In what direction is the directional derivative the greatest? What is the directional derivative in that direction?
d) Suppose the surface $z = \sqrt{x} + y$ represents a mountain (x, y, z in kilometers), and assume a climber is passing through point $(9, -2, 1)$ so that the horizontal component of her motion is in direction of $\langle 1, -2 \rangle$ with unit speed (time in hours). Is she ascending or descending, and at which rate? What are the units?

2. (12pts) Find the equation of the tangent plane to the hyperboloid of one sheet $x^2 + 2y^2 - z^2 = 23$ at the point $(4, -2, -1)$. Simplify the equation to standard form.

3. (18pts) Let $K = ye^{-x^2-y^2}$, $x = \dfrac{\sin u}{\cos v}$, $y = \cos u + \sin v$. Use the chain rule to find $\dfrac{\partial K}{\partial v}$ when $u = \frac{\pi}{2}$, $v = \frac{\pi}{4}$.

4. (14pts) A museum is housed in a cylindrical building (with a flat roof) with interior diameter 40 meters and height 30 meters. Use differentials to estimate the amount of concrete used to build this museum, if its walls have thickness 0.3 meters and the roof has thickness 0.1 meter.

5. (14pts) Use implicit differentiation to find $\dfrac{\partial z}{\partial y}$ at the point $(3, -2, 4)$, if $x \ln(x^2 + yz) + y\sqrt{7z - 4x} = -8$.

6. (20pts) Find and classify the local extremes for $f(x, y) = x^3 - 12xy + 8y^3$.

Bonus (10pts) Let $A = (0,0)$ and $B = (0,1)$, and let d_A and d_B represent the distance from a point (x,y) to A and B, respectively. Find the absolute maximum and minimum of $d_A^2 + d_B^2$ among all points (x,y) in the upper half of the unit disk $x^2 + y^2 \leq 1$, $y \geq 0$.

Calculus 3 — Exam 3 **MAT 309, Fall 2016 — D. Ivanšić**

Name: ______________________________

Show all your work!

1. (18pts) Find $\iint_D \frac{1}{x}\, dA$ if D is the triangle bounded by lines $x = 1$, $y = x - 1$ and $y = 7 - 3x$. Sketch the region of integration first.

2. (18pts) Let D be the region bounded by the curves $y = x^3$, $x = 2$ and $y = 0$. Sketch the region and set up $\displaystyle\iint_D \frac{1}{(x^4+1)^2}\, dA$ as iterated integrals in both orders of integration. Then evaluate the double integral using the easier order. Sketch the region of integration first.

3. (18pts) Use polar coordinates to find $\displaystyle\iint_D \frac{x}{x^2+y^2}\,dA$, if D is the region inside the circle $(x-1)^2+y^2=1$ and outside the circle $x^2+y^2=1$. Sketch the region of integration first.

4. (18pts) Sketch the region E that is bounded by the planes $z=0$, $z=\frac{x}{3}$ and the parabolic cylinder $x=9-y^2$. Then write the two iterated triple integrals that stand for $\iiint_E f\,dV$ which end in $dy\,dz\,dx$ and $dx\,dz\,dy$.

5. (10pts) Find the cylindrical and spherical coordinates of the point whose cartesian coordinates are $(-\sqrt{6}, \sqrt{6}, 2)$.

6. (18pts) Use either cylindrical or spherical coordinates to set up $\iiint_E x^2 + z^2 \, dV$, where E is the region inside the sphere $x^2 + y^2 + z^2 = 16$ and above the cone $x^2 + y^2 = z^2$. Simplify the expression but do not evaluate the integral. Sketch the region E.

Bonus (10pts) Find the volume of the solid that is inside both cylinders: $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.

Calculus 3 — Exam 4 **MAT 309, Fall 2016 — D. Ivanšić**

Name: ______________________________

Show all your work!

1. (12pts) Let $\mathbf{F}(x,y) = \left\langle -\frac{x}{2}, -\frac{2y}{9} \right\rangle$.
a) Guess a function $f(x,y)$ so that $\mathbf{F} = \nabla f$.
b) Use the function f to draw the vector field without having to evaluate F at various points.

2. (20pts) In both cases set up and simplify the set-up, but do NOT evaluate the integral.
a) $\int_C \dfrac{y}{x^2+y^2+z^2}\, ds$, where C is the helix $x = 3\cos t$, $y = \frac{t}{\pi}$, $z = 3\sin t$, t in $[0, 4\pi]$.
b) $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x,y) = \left\langle x^2y^2, \dfrac{x}{y} \right\rangle$, where C is the arc of the hyperbola $x^2 - y^2 = 1$ from point $(2, \sqrt{3})$ to point $(5, 2\sqrt{6})$.

3. (10pts) Let $f(x, y) = x^2 + xy + y^2$, and let $\mathbf{F} = \nabla f$. Apply the fundamental theorem for line integrals to answer:
a) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the part of the right half of the circle $x^2 + (y-2)^2 = 4$ from $(0, 0)$ to $(1, 2+\sqrt{3})$? How about if C is a straight line segment from $(0, 0)$ to $(1, 2+\sqrt{3})$?
b) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the curve consisting of the left half of the circle from a) together with the line segment from $(0, 0)$ to $(0, 4)$, traversed clockwise?

4. (22pts) a) Find curl of both of the vector fields below.
b) One of the fields is conservative. Find its potential function.

$\mathbf{F}(x, y, z) = \langle \sin z, x, x\cos z - \sin z \rangle$ $\qquad$ $\mathbf{G}(x, y, z) = \langle 3x^2y^2, 2x^3y + e^z, (2+y)e^z \rangle$

5. (24pts) Use Green's theorem to find the line integral $\int_C (x^2 + y^2)\, dx + xy\, dy$, where C is the boundary of the trapezoid with vertices $(1, 0)$, $(3, 0)$, $(3, 7)$ and $(1, 1)$, traversed counterclockwise. Draw the trapezoid.

6. (12pts) Use Green's theorem to find the area of the unit disk.

Bonus. (10pts) Recall that we have shown that the field $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$, defined on the region $D = \mathbf{R}^2$ without the origin, is not conservative on D, since its line integral over a unit circle is not 0.

a) Let $f(x, y) = \arctan \frac{y}{x}$. Show that $\nabla f = \mathbf{F}$. Recall that $(\arctan u)' = \frac{1}{1+u^2}$.

b) Why does a) not contradict our earlier finding of $\mathbf{F}$ not being conservative on D?

Calculus 3 — Final Exam **MAT 309, Fall 2016 — D. Ivanšić**

Name: ______________________________

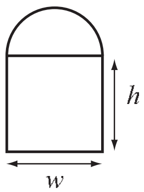
Show all your work!

1. (12pts) Find the equation of the plane whose intersections with the x- and z-axes are 3 and 5, respectively, and that contains the point $(2, 3, -2)$.

2. (22pts) Let $f(x, y) = xy$.
a) Sketch the contour map for the function, drawing level curves for levels $k = -2, -1, 0, 1, 2$.
b) At point $(3, -4)$, find the directional derivative of f in the direction of $\langle 2, -1\rangle$. In what direction is the directional derivative the greatest? What is the directional derivative in that direction?
c) Let $\mathbf{F} = \nabla f$. Sketch the vector field $\mathbf{F}$.
d) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the straight line from $(0, 0)$ to $(5, 2)$?
e) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is any part of one level curve?

3. (20pts) Find and classify the local extremes for $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

4. (12pts) A Norman window is a rectangle topped by a semicircle. Use differentials to estimate how much the area of the window changes if the height increases from 52 to 56 inches, and the width decreases from 40 to 38 inches.



5. (18pts) Let D be the region bounded by the curves $y = x^2$, $x = 0$ and $y = 9$. Sketch the region and set up $\displaystyle\iint_D \frac{1}{y^{\frac{3}{2}} + 1}\, dA$ as iterated integrals in both orders of integration. Then evaluate the double integral using the easier order. Sketch the region of integration first.

6. (18pts) Sketch the region E that is bounded by the planes $z = 0$, $x = 9$, $z = 3 - y$ and the parabolic cylinder $x = 9 - y^2$. Then write the two iterated triple integrals that stand for $\iiint_E f\, dV$ which end in $dz\, dy\, dx$ and $dy\, dx\, dz$.

7. (16pts) Use either cylindrical or spherical coordinates to set up $\displaystyle\iiint_E \frac{x+y}{x^2+y^2+z^2}\, dV$, where E is the region inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2+y^2 = 8$. Simplify the expression but do NOT evaluate the integral. Sketch the region E.

8. (10pts) Set up and simplify the set-up, but do NOT evaluate the integral: $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x, y) = \langle xy, x + y^2 \rangle$, and C is the arc of the circle $x^2 + y^2 = 8$ from point $(-1, \sqrt{7})$ to point $(2, 2)$.

9. (22pts) Use Green's theorem to find the line integral $\int_C (x^2 + y^2)\, dx + xy\, dy$, where C is the boundary of the triangle with vertices $(0, 0)$, $(2, 6)$ and $(0, 12)$, traversed counterclockwise. Draw the triangle.

Bonus (15pts) Let $A = (0,0)$ and $B = (0,1)$, and let d_A and d_B represent the distance from a point (x, y) to A and B, respectively. Find the absolute maximum and minimum of $d_A^2 + d_B^2$ among all points (x, y) in the upper half of the unit disk $x^2 + y^2 \leq 1$, $y \geq 0$.