Calculus 3 — Final Exam
MAT 309, Fall 2013 — D. Ivanšić

Name: _____

Show all your work!

1. (12pts) Find the equation of the plane that contains the point $(3, 2, -4)$ and the line $x = 1 - 2t$, $y = 3 + t$, $z = -2 - t$.

2. (16pts) Let $f(x, y) = x^2 - y^2$.
a) Sketch the contour map for the function, drawing level curves for levels $k = -2, -1, 0, 1, 2$.
b) At point $(2, -1)$, what is the directional derivative of f in the direction of $\langle 1, -1 \rangle$?
c) In what direction is the directional derivative of f the greatest at $(2, -1)$, and what is it?
d) If C is the curve parametrized by $x = 2t - 1$, $y = t^2 + 3t$, $0 \leq t \leq 2$, what is $\int_C \nabla f \cdot d\mathbf{r}$?

3. (10pts) Find the equation of the tangent plane to the surface $\frac{x^2}{9} - \frac{y^2}{4} - \frac{z^2}{16} = 1$ at the point $(6, -2\sqrt{2}, 4)$. Simplify the equation to standard form.

4. (12pts) The volume of a cylinder is given by $V = \pi r^2 h$. When $r = 2$ meters and $h = 5$ meters, use differentials to estimate the change in volume of the cylinder, if its radius decreases by 0.1m and its height increases by 0.2m.

5. (16pts) Let D be the region bounded by the curves $x = 0$, $y = \frac{1}{3}$ and $y = e^x$. Sketch the region and set up $\iint_D y^2\,dA$ as iterated integrals in both orders of integration. Then evaluate the double integral using the easier order.

6. (18pts) Find and classify the local extremes for $f(x, y) = x^3 + 3x^2y - y^3 + 9y$.

7. (16pts) Use cylindrical or spherical coordinates to find $\iiint_E x^2 + y^2\, dV$, where E is the region inside the sphere $x^2 + y^2 + z^2 = 9$ and above the cone $z = \sqrt{3(x^2 + y^2)}$ Sketch the region E.

Bonus (10pts) A ray of light, represented by the line $x = 2 - t$, $y = 4 + 2t$, $z = -3 - 3t$ reflects off the mirror represented by the plane $x - y + 2z = 10$ at point $P = (4, 0, 3)$. Find parametric equations of the line that represents the reflected ray. *(Hints: the ray and the reflected ray determine a plane that is perpendicular to the mirror. Vector projection is helpful.)*