Calculus 3 — Exam 5
MAT 309, Fall 2013 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (10pts) Let $f(x,y) = \dfrac{x^3}{y^2}$, and let $\mathbf{F} = \nabla f$. Apply the fundamental theorem for line integrals to answer:

a) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is part of the parabola $y = x^2$ from $(1,1)$ to $(3,9)$? How about if C is a straight line segment from $(1,1)$ to $(3,9)$?

b) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the circle centered at $(3,4)$ with radius 2?

2. (12pts) Find curl $\mathbf{F}$ and div $\mathbf{F}$ if $\mathbf{F}(x,y,z) = \langle z^2 - 4y^2, 4x^2 - 3z^2, 3y^2 - x^2 \rangle$.

3. (14pts) A surface is parametrized by $\mathbf{r}(u,v) = \langle u^2, v^2, u+v \rangle$. Find the equation of the tangent plane to this surface at the point where $(u,v) = (2,-3)$.

4. (20pts) One of the two vectors fields below is not a gradient field, and the other one is (curl detects it). Identify which is which, and find the potential function for the one that is.

$\mathbf{F}(x,y,z) = \langle 2x\sin z + y^3 e^x, 3y^2 e^x + \cos z, x^2 \cos z - y \sin z \rangle$ $\qquad$ $\mathbf{G}(x,y,z) = \langle x^2, y^2, yz^2 \rangle$

5. (26pts) Consider the part of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1$ between the planes $z = -2$ and $z = 0$.

a) Draw the surface, parametrize it and specify the planar region D where your parameters come from.

b) Set up the iterated integral that gives the area of the surface. Simplify the set-up, but do not evaluate the integral.

6. (18pts) Use Green's theorem to find the line integral $\int_C x^3\, dx + xy\, dy$, where C is the triangle from $(-1, 0)$ to $(1, 0)$ to $(0, 1)$ to $(-1, 0)$.

Bonus. (10pts) Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = 4$ that is above the line $y = 1$.