Calculus 3 — Exam 4 **MAT 309, Fall 2013 — D. Ivanšić**

Name: ______________________________

Show all your work!

1. (18pts) Use cylindrical coordinates to find the volume of the region E enclosed by the paraboloids $z = x^2 + y^2$ and $z = 3 - \frac{1}{2}(x^2 + y^2)$. Sketch the region E.

2. (18pts) Use spherical coordinates to find $\iiint_E xz\, dV$, where E is the part of the first octant that is inside the sphere $x^2 + y^2 + z^2 = 16$, and outside the sphere $x^2 + y^2 + z^2 = 9$. Sketch the region E.

3. (14pts) Let $\mathbf{F}(x,y) = \langle x, 3\rangle$.
a) Roughly draw the vector field $\mathbf{F}(x,y)$, scaling the vectors for a better picture.
b) Guess a function $f(x,y)$ so that $\mathbf{F} = \nabla f$.
c) How could you have roughly done a) without evaluating the vector field at various points?

4. (18pts) In both cases set up and simplify the set-up, but do not evaluate the integral.
a) $\int_C x^2 - y^2 + z^2\, ds$, where C is the line segment from $(0,0,1)$ to $(1,-3,3)$.
b) $\int_C \mathbf{F}\cdot d\mathbf{r}$, if $\mathbf{F}(x,y) = \langle xe^y, ye^x\rangle$, where C is the circle of radius 5 centered at the origin.

5. (12pts) Find the cylindrical and spherical coordinates of the point whose cartesian coordinates are $(-\sqrt{6}, -\sqrt{2}, 2\sqrt{2})$.

6. (20pts) Use change of variables to find $\iint_D y\, dA$, if D is the rectangle that is bounded by the lines $y = x$, $y = x + 5$, $y = -x$, $y = -x + 1$. Sketch the rectangle.

Bonus. (10pts) Find the Jacobian $\dfrac{\partial(x,y,z)}{\partial(\rho,\theta,\Phi)}$, where x, y, z are functions that convert spherical coordinates to cartesian. What do you expect to get?