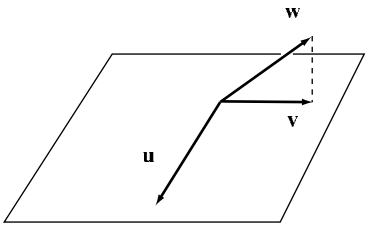
**Calculus 3 — Exam 1**
**MAT 309, Fall 2013 — D. Ivanšić**

**Name:** _____

*Show all your work!*

**1.** (16pts) Let $\mathbf{u} = \langle 1, 1, -4 \rangle$ and $\mathbf{v} = \langle 3, -1, 0 \rangle$.
a) Calculate $3\mathbf{u}$, $2\mathbf{u} - 4\mathbf{v}$, and $|\mathbf{u}|$.
b) Find the unit vector in direction of $\mathbf{v}$.
c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

**2.** (12pts) In the picture, vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular and the projection of vector $\mathbf{w}$ to the vector $\mathbf{v}$ is the vector $\mathbf{v}$. The angle between vectors $\mathbf{v}$ and $\mathbf{w}$ is $\pi/6$. Suppose that $|\mathbf{u}| = 7$, $|\mathbf{v}| = 3$ and $|\mathbf{w}| = 2\sqrt{3}$. Draw the vector $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ in the picture. What is its length?



**3.** (8pts) Draw the region in $\mathbf{R}^3$ described by:

$\frac{x^2}{9} + \frac{z^2}{4} = 1,\ x \geq 0$

**4.** (6pts) Which of the following expressions are meaningful? Briefly explain.

$\mathbf{u} \times (\mathbf{v} \cdot \mathbf{u})$ $(\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \cdot \mathbf{w})$ $(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \times \mathbf{v})$

**5.** (20pts) A parallelogram in $\mathbf{R}^3$ has vertices $A = (3, 1, 4)$, $B = (5, 2, 4)$, $C = (8, 1, 3)$, and $D = (6, 0, 3)$.
a) Find the equation of the plane that contains this parallelogram.
b) Find the area of the parallelogram.
c) Is this parallelogram a rectangle?

**6.** (22pts) The curve $\mathbf{r}(t) = \langle (t+1)\cos t, (t+1)\sin t, 2t \rangle$ is given, $0 \leq t \leq 4\pi$.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = 2\pi$ and sketch the tangent line.
c) Set up the integral for the length of the curve. Simplify the function inside the integral as much as possible, but do not evaluate the integral.

**7.** (16pts) This problem is about the surface $\frac{x^2}{4} - \frac{y^2}{25} - \frac{z^2}{16} = 1$.
a) Identify and sketch the intersections of this surface with the coordinate planes.
b) Sketch the surface in 3D, with coordinate system visible.

**Bonus** (10pts) A ray of light, represented by the line $x = 2 - t$, $y = 4 + 2t$, $z = -3 - 3t$ reflects off the mirror represented by the plane $x - y + 2z = 10$ at point $P = (4, 0, 3)$. Find parametric equations of the line that represents the reflected ray. *(Hints: the ray and the reflected ray determine a plane that is perpendicular to the mirror. Vector projection is helpful.)*