## College Algebra - Joysheet 1

MAT 140, Fall 2016 - D. Ivanšić

Name:
Covers: 1.1, 1.2 Show all your work!

1. (10pts) Draw the points $A=(-2,0), B=(1,4)$ and $C=(9,-1)$.
a) Which of points $A$ or $B$ is closer to $C$ ?
b) Is the triangle $A B C$ is a right triangle?
2. (8pts) Write the equation of the circle whose diameter has endpoints $(3,-4)$ and $(-1,0)$. Sketch the circle.
3. (8pts) Use the graph of the function $f$ at right to answer the following questions.
a) Find $f(2)$ and $f(1.5)$.
b) What is the domain of $f$ ?
c) What is the range of $f$ ?
d) What are the solutions of the equation $f(x)=-3$ ?

4. (12pts) The function
$f(x)=x^{4}-7 x^{3}+x^{2}-x-5$ is given.
a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.
b) Find all the $x-$ and $y$-intercepts (accuracy: 6 decimal points).
c) State the domain and range.
5. (12pts) Find the domain of each function and write it using interval notation.
$g(x)=\frac{\sqrt{x}}{4 x-15}$

$$
f(x)=\frac{|2 x-3|}{x^{2}-5 x-14}
$$

6. (10pts) Let $g(x)=\frac{x^{2}+2 x-3}{2 x-8}$. Find the following (simplify where appropriate).
$g(1)=$

$$
g(4)=
$$

$g(-3 t)=$

$$
g(u+4)=
$$

College Algebra - Joysheet 2
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## Name:

Covers: 1.3, 1.4 Show all your work!

1. (10pts) Find the equation of the line (in form $y=m x+b$ ) that passes through point $(1,4)$ and is parallel to the line $4 x-5 y=1$. Draw both lines.
2. (5pts) Find the linear function $f$ its $x$-intercept is -4 and its $y$-intercept is 1 .
3. (9pts) Draw the quadrangle with vertices $A=(5,6), B=(-6,7), C=(-3,-6)$ and $D=(7,-4)$. Use slopes to determine if the any of the angles in the quadrangle is a right angle.
4. (4pts) Price of milk varies with season. In January, the average U.S. price of a gallon of milk was $\$ 3.313$. In July, it was $\$ 3.062$. What is the average rate of change of the price of milk from January to July? What are the units for the average rate of change?
5. (12pts) A family keeps track of their electric bill. One month, they used 1258 kilowatthours and paid $\$ 174.73$. Another month, they used 980 kilowatt-hours and paid $\$ 139.98$.
a) Assuming that the cost of electricity $C(x)$ is a linear function of kilowatt-hours $x$ used, write a formula for $C(x)$.
b) How much would they pay for 0 kilowatt-hours used? What is the meaning of this number? c) What is the meaning of the slope in this example?
6. (20pts) A clothing store manager is trying to establish the relationship between the price of a pair of jeans and the weekly number of jeans sold. The table shows the data: $P$ is the price in dollars and $S$ is the number of jeans sold by the store in a week. Solve the problems below with accuracy 6 decimal points.
a) Draw the scatterplot of the data. Does the relationship look linear?
b) Use two points in the scatterplot to get an equation of a line that models the relationship between $P$ and $S$. Draw the line on the graph.
c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.
d) Find the coefficient of correlation $r$. How strong is the linear relationship between $P$ and $S$ ?
e) What amount of sales does the manager expect if the price is set at $\$ 64$ ?

| $P$ | $S$ |
| :---: | :---: |
| 39 | 180 |
| 49 | 142 |
| 54 | 126 |
| 57 | 120 |
| 69 | 55 |
| 76 | 27 |
| 79 | 18 |

## College Algebra - Joysheet 3 MAT 140, Fall 2016 - D. Ivanšić

Name:
Covers: 1.5, 1.6 Show all your work!

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-1 \leq 2 x-7<6$
2. (7pts) $2 x+3<5$ or $3 x-7>11$
3. (6pts) Find the domain of the function in interval notation: $f(x)=\frac{\sqrt{10-3 x}}{2 x-5}$.
4. (14pts) Luciana has $\$ 8,000$ to invest and can split this money between an investment bringing $4 \%$ interest, and one bringing $3 \%$ interest. What is the least she needs to invest at $4 \%$ interest in order to meet a goal of annual interest of at least $\$ 290$ ?
5. (14pts) Paul traveled to Lexington at 70 miles per hour. On the way back, he took the scenic route and averaged 55 miles per hour. The way back was 86 miles longer and took 2.6 hours longer to drive than the way to Lexington.
a) How long did it take to drive to Lexington?
b) How long is the scenic route from Lexington?
6. (14pts) How many liters of water must be mixed with 4 liters of a $22 \%$ solution of muriatic acid in order to get a $12 \%$ solution of muriatic acid?

## College Algebra - Joysheet 4 MAT 140, Fall 2016 - D. Ivanšić

## Name:

Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x)=\frac{5 x+7}{x^{2}+4}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.
2. (20pts) Let $f(x)=3 \sqrt{x}-1, g(x)=\frac{2 x-1}{x^{2}+x-6}$. Find the following (simplify where possible):
$(f+g)(4)=\quad(f g)(8)=$
$\frac{f}{g}(x)=$
$(g \circ f)(9)=$
$(f \circ g)(x)=$

The domain of $\frac{f}{g}(x)$ in interval notation
3. (8pts) Consider the function $h(x)=(4 x-7)^{2}$ and find two different solutions to the following problem: find functions $f$ and $g$ so that $h(x)=f(g(x))$, where neither $f$ nor $g$ are the identity function.
4. (8pts) Sketch the graph of the piecewise-defined function:
$f(x)= \begin{cases}-2 x-7, & \text { if } x \leq-2 \\ 3-x, & \text { if }-2<x \leq 5 .\end{cases}$
5. (14pts) A retail chain with two brands is opening two new stores, to be housed in the same building with total area 4000 square feet. The two stores share a wall and have 5 - ft openings on the corners to allow for entrances (see picture). The chain wishes to minimize the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides $x$. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the building for which the total length of the walls is minimal? What is the minimal wall length?


## College Algebra - Joysheet 5 <br> MAT 140, Fall 2016 - D. Ivanšić

## Name:

Covers: 2.4, 2.5 Show all your work!

1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.
$f(x)=x^{3}-x+4$
$g(x)=x^{5}-13 x^{3}+36$
$h(x)=x^{2}-4|x|-2$
2. (16pts) Using transformations, draw the graphs of $f(x)=-\sqrt{x+4}$ and $g(x)=\frac{2}{-x}+5$. Explain how you transform graphs of basic functions in order to get the graphs of $f$ and $g$.
3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y=x^{3}$, shifted right 4 units
b) shape of $y=\frac{1}{x}$ stretched horizontally by factor 4 , then shifted up 3 units
c) shape of $y=|x|$, stretched vertically by factor 2 , then reflected about the $x$-axis, then shifted down 1 unit.
4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x)+3,2 f(x-1)$ and $-f(2 x)$ and label all the relevant points.


## College Algebra - Joysheet 6 MAT 140, Fall 2016 - D. Ivanšić

Name:
Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a+b i$.

1. $(4 \mathrm{pts}) 3+i+(2-3 i)^{2}=$
2. $(6 \mathrm{pts}) \frac{4+3 i}{5-2 i}=$
3. (4pts) Simplify and justify your answer.
$i^{214}=$
4. (8pts) The number of crates of apples in storage of an apple grower is described by the function $C(x)=-x^{2}+45 x+16$, where $x$ is the number of days after September 1st. On what dates did the apple grower have 390 crates in storage?
5. (8pts) Solve the equation: $3 x^{4}-7 x^{2}-20=0$
6. ( 6 pts ) Solve by completing the square.
$x^{2}-16 x+19=0$
7. (12pts) The quadratic function $f(x)=-2 x^{2}+5 x-4$ is given. Do the following without using the calculator.
a) Find the $x$-intercepts of its graph, if any. Find the $y$-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
8. (12pts) Donald's house sits on a big rectangular plot of land that is 150 by 70 yards. He wishes to enlarge it to get a rectangular plot with area 20,000 square yards by extending the 150 -yard side by a certain amount and increasing the 70 -yard side by twice that amount. By how much should the 150 - and 70 -yard sides be extended to achieve the desired area?

## College Algebra - Joysheet 7 <br> MAT 140, Fall 2016 - D. Ivanšić

Name:
Covers: 3.3, 3.4, 3.5 Show all your work!

1. (4pts) Solve the equation.
$|4 x+3|=5$
2. (12pts) Solve the inequalities. Draw your solution and write it in interval form. $|x+7| \geq 2$

$$
|3 x-4|<6
$$

Solve the equations:
3. (8pts) $\frac{x}{x-5}-\frac{30}{x^{2}-4 x-5}=\frac{-4}{x+1}$
4. $(8 \mathrm{pts}) 2 x+3=x+\sqrt{3 x+37}$
5. (14pts) A ball is thrown upwards from the ground with initial velocity 17 meters per second. Its height in meters after $t$ seconds is given by $s(t)=-5 t^{2}+17 t$.
a) Sketch the graph of the height function.
b) When does the ball reach its greatest height, and what is that height?
c) When is the ball at height 12 meters?
6. (14pts) A farm is fencing in a rectangular area along a river and dividing it in three sections (see picture). The part along the river is not fenced. The farm budgeted for 10,000 feet of fence, and its goal is to maximize the total enclosed area.
a) Express the total area of the fenced-in field as a function of one of the sides of the rectangle. What is the domain of this function?
b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator - you should already know what the graph looks like). What are the dimensions of the fenced-in field that has the greatest total area and what is the greatest total area possible?


## College Algebra - Joysheet 8 <br> MAT 140, Fall 2016 - D. Ivanšić

1. (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points.

Name:
Covers: 5.1-5.3 Show all your work!

2. (12pts) Let $f(x)=\frac{3 x+2}{x-5}$. Find the formula for $f^{-1}$. Find the ranges of $f$ and $f^{-1}$.
3. (8pts) Evaluate without using the calculator:
$\log _{6} 36=\quad \log _{3} \frac{1}{81}=\quad \log _{16} 4=\quad \log _{b} \sqrt[3]{b^{4}}=$
4. (4pts) Use your calculator to find $\log _{12} 49$ with accuracy 6 decimal places. Show how you obtained your number.
5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of $\$ 4,000$ and annual interest rate of $3 \%$, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year and enter them below.
c) Does compounding more often make a big difference?

| Frequency: every | $n$ | Amount after 1 year |
| :---: | :---: | :---: |
| year |  |  |
| quarter |  |  |
| month |  |  |
| day |  |  |
| hour |  |  |

6. (3pts) Find the domain of $f(x)=\log _{7}(4+5 x)$.
7. (6pts) Students took equivalent forms of the same exam at monthly intervals. The average score $S(t)$ (in percent) was found to be given by the function $S(t)=78-15 \log (t+1)$, where $t$ is the number of months after the initial exam.
a) What was the average score on the inital exam? After 5 months? After 15 months? (Round to whole numbers.)
b) Use the intersect feature on the calculator to estimate after how many months is the average score $55 \%$ ?
8. (9pts) Using transformations, draw the graph of $f(x)=5+\frac{1}{2} \cdot 3^{-x}$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Show at least one point on the graph, and asymptotes to the graph, if any.

## College Algebra - Joysheet 9 <br> MAT 140, Fall 2016 - D. Ivanšić

Name:
Covers: 5.4-5.6 Show all your work!

1. (5pts) If $\log _{a} 4=0.548869$ and $\log _{a} 7=0.770435$, find (show how you obtained your numbers):
$\log _{a} 28=$ $\log _{a} \frac{4}{49}=$
2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\log _{6}\left(216 x^{4} y^{7}\right)=$
$\log \frac{1000 x^{2} y^{\frac{3}{2}}}{\sqrt{x^{5} y}}=$
3. (12pts) Write as a single logarithm. Simplify if possible.
$\frac{1}{4} \ln \left(16 x^{8}\right)-3 \ln \left(2 y^{\frac{5}{6}}\right)-2 \ln x=$
$3 \log _{7}\left(x^{2}-12 x+32\right)-2 \log _{7}(x-4)-\log _{7}(x-8)=$

Solve the equations.
4. $(5 \mathrm{pts}) 27^{4 x+5}=\left(\frac{1}{9}\right)^{1-2 x}$
5. $(7 \mathrm{pts}) 6^{4 x+1}=2^{x-5}$
6. $(8 \mathrm{pts}) \log _{9}(x-5)+\log _{9}(x+3)=1$
7. (12pts) According to US census data, Indianapolis, IN, had 781,926 inhabitants in 2000 and 820,445 in 2010 . Assume the population of Indianapolis grows exponentially.
a) Write the function describing the number $P(t)$ of people in Indianapolis $t$ years after 2000 . Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach $1,000,000$ ?

